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## **random excitation memory-dependent hydrodynamics and Capsize criteria for ship models with**

C. Jiang, A. W. Troesch and S. W. Shaw

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## Capsize criteria for ship models with Capsize criteria for ship models with<br>memory-dependent hydrodynamics and<br>random excitation riteria for ship models<br>ependent hydrodynam<br>random excitation<br>Alberta **random excitation**<br>By C. JIANG<sup>1</sup>, A. W. TROESCH<sup>2</sup> AND S. W. SHAW<sup>3</sup>

BY C. JIANG<sup>1</sup>, A. W. TROESCH<sup>2</sup> AND S. W. SHAW<sup>3</sup><br><sup>1</sup>*Band, Lavis and Associates, Severna Park, MD 21146, USA*<br><sup>2</sup> Department of Naval Architecture and Marine Engineering BY C. JIANG<sup>1</sup>, A. W. TROESCH<sup>2</sup> AND S. W. SHAW<sup>3</sup><br><sup>2</sup> Department of Naval Architecture and Marine Engineering,<br><sup>2</sup> Department of Naval Architecture and Marine Engineering, *Band, Lavis and Associates, Severna Park, MD 21146, USA*<br> *Department of Naval Architecture and Marine Engineering,*<br> *University of Michigan, Ann Arbor, MI 48109-2145, USA*<br> *extragnt of Mechanical Engineering, Michigan* <sup>3</sup>*Department of Mechanical Engineering, Michigan State University, Eniversity of Michigan, Ann Arbor, MI 48109-2145, USA* 

The large-amplitude rolling and capsize dynamics of vessels in random beam seas are<br>investigated using a nonlinear single-degree-of-freedom model Included in this model The large-amplitude rolling and capsize dynamics of vessels in random beam seas are<br>investigated using a nonlinear single-degree-of-freedom model. Included in this model<br>are three types of damping moments—the usual effects The large-amplitude rolling and capsize dynamics of vessels in random beam seas are<br>investigated using a nonlinear single-degree-of-freedom model. Included in this model<br>are three types of damping moments—the usual effects investigated using a nonlinear single-degree-of-freedom model. Included in this model<br>are three types of damping moments—the usual effects that are treated as linear<br>and quadratic in the roll velocity, plus a frequency-dep ō are three types of damping moments—the usual effects that are treated as linear<br>and quadratic in the roll velocity, plus a frequency-dependent effect that captures<br>the dissipation of energy caused by the generation of wave and quadratic in the roll velocity, plus a frequency-dependent effect that captures<br>the dissipation of energy caused by the generation of waves radiated away from the<br>rolling vessel. The description of this type of damping the dissipation of energy caused by the generation of waves radiated away from the rolling vessel. The description of this type of damping requires a history-dependent term in the equations of motion. This memory effect pr rolling vessel. The description of this type of damping requires a history-dependent<br>term in the equations of motion. This memory effect prevents a straightforward<br>application of the standard Melnikov method for determinin term in the equations of motion. This memory effect prevents a straightforward application of the standard Melnikov method for determining capsize criteria. In this work, the Melnikov function and phase-space transport tec application of the standard Melnikov method for determining capsize criteria. In this work, the Melnikov function and phase-space transport techniques are extended<br>to derive a criterion for capsizing that can be applied to analytical models with this<br>type of damping. Using these theoretical results, we to derive a criterion for capsizing that can be applied to analytical models with this type of damping. Using these theoretical results, we obtain a closed-form asymptotic expression for a critical significant wave height, type of damping. Using these theoretical results, we obtain a closed-form asymptotic

S for a realistic set of vessel parameters.<br>Keywords: ship capsize; nonlinear random processes;<br>slobal stability: Melnikov method global stability; Melnikov method<br>global stability; Melnikov method

## 1. Introduction

The nonlinear behaviour of ship motion leading to capsize has been extensively stud-<br>ied by many authors using a variety of mathematical models for the ship and the sea<br>state. The dynamic models proposed for ship dynamics The nonlinear behaviour of ship motion leading to capsize has been extensively stud-The nonlinear behaviour of ship motion leading to capsize has been extensively stud-<br>ied by many authors using a variety of mathematical models for the ship and the sea<br>state. The dynamic models proposed for ship dynamics ied by many authors using a variety of mathematical models for the ship and the sea<br>state. The dynamic models proposed for ship dynamics range from those describing<br>the full six degrees of freedom to those with only a sing state. The dynamic models proposed for ship dynamics range from those describing<br>the full six degrees of freedom to those with only a single degree of freedom (typi-<br>cally roll). Examples of research using multi-degree-of the full six degrees of freedom to those with only a single degree of freedom (typically roll). Examples of research using multi-degree-of-freedom models are given by Vassalos & Spyrou (1990), Falzarano & Zhang (1993), Um cally roll). Examples of research using multi-degree-of-freedom models are given by<br>Vassalos & Spyrou (1990), Falzarano & Zhang (1993), Umeda & Renilson (1994),<br>Spyrou & Umeda (1995), Spyrou (1996) and Vassalos *et al.* (1  $\sqrt{ }$ Vassalos & Spyrou (1990), Falzarano & Zhang (1993), Umeda & Renilson (1994),<br>Spyrou & Umeda (1995), Spyrou (1996) and Vassalos *et al.* (1999). A reduction in<br>the degrees of freedom results in more idealized models, and o Spyrou & Umeda (1995), Spyrou (1996) and Vassalos *et al.* (1999). A reduction in the degrees of freedom results in more idealized models, and often allows for a more rigorous analysis that yields mathematical results in the degrees of freedom results in more idealized models, and often allows for a more rigorous analysis that yields mathematical results in terms of the system parameters. Such reductions can, in many cases, be mathematical rigorous analysis that yields mathematical results in terms of the system parame-<br>ters. Such reductions can, in many cases, be mathematically justified using invariant<br>manifold theory, as demonstrated by Chen *et al.* (199 ters. Such reductions can, in many cases, be mathematically justified using invariant<br>manifold theory, as demonstrated by Chen *et al.* (1999). The work described in this<br>paper is based upon such a single-degree-of-freedo manifold theory, as demonstrated by Chen *et al.* (1999). The work described is paper is based upon such a single-degree-of-freedom (SDOF) model, represent rolling vessel in random beam seas with memory-dependent hydrodyn rolling vessel in random beam seas with memory-dependent hydrodynamics.<br>*Phil. Trans. R. Soc. Lond.* A (2000) 358, 1761–1791 (2000) The Roya

Many methods have been developed to analyse nonlinear ship rolling for SDOF models. Examples of the more successful of these are the regular perturbation method (Wright & Marshfield 1980: Cardo *et al.* 1981, 1984), the m **MATHEMATICAL,<br>PHYSICAL<br>& ENGINEERING<br>SCIENCES** Many methods have been developed to analyse nonlinear ship rolling for SDOF models. Examples of the more successful of these are the regular perturbation method (Wright & Marshfield 1980; Cardo *et al.* 1981, 1984), the mu Many methods have been developed to analyse nonlinear ship rolling for SDOF models. Examples of the more successful of these are the regular perturbation method (Wright & Marshfield 1980; Cardo *et al.* 1981, 1984), the multiple scales method (Nayfeh & Khdeir 1986*a,b*), the method of averaging (N (Wright & Marshfield 1980; Cardo *et al.* 1981, 1984), the multiple scales method (Nayfeh & Khdeir 1986*a,b*), the method of averaging (Nayfeh 1973; Cardo *et al.* 1981, 1984), the harmonic balance method (Wright & Marshf (Nayfeh & Khdeir 1986*a*,*b*), the method of averaging (Nayfeh 1973; Cardo *et al.* 1981, 1984), the harmonic balance method (Wright & Marshfield 1980; Senjanović 1994) and numerical simulation methods (Thompson 1989*a*, 1984), the harmonic balance method (Wright & Marshfield 1980; Senjanović 1994) and numerical simulation methods (Thompson 1989 $a, b, 1990$ ; Virgin 1987, 1989). The first three methods are restricted to weakly nonlinear sys and numerical simulation methods (Thompson 1989 $a, b$ , 1990; Virgin 1987, 1989).<br>The first three methods are restricted to weakly nonlinear systems, which are not<br>representative of large-amplitude roll motion that can lead The first three methods are restricted to weakly nonlinear systems, which are not representative of large-amplitude roll motion that can lead to capsize. In order to achieve convergence for the harmonic balance method, man representative of large-amplitude roll motion that can lead to capsize. In order to achieve convergence for the harmonic balance method, many terms are needed, and this often makes the resulting algebraic equations prohibi achieve convergence for the harmonic balance method, many terms are needed, and<br>this often makes the resulting algebraic equations prohibitively complicated. Simu-<br>lation results are very powerful, especially when combined this often makes the resulting algebraic equations prohibitively complicated. Simulation results are very powerful, especially when combined with an understanding of the underlying phase space, as done in the ground-break lation results are very powerful, especially when combined with an understanding<br>of the underlying phase space, as done in the ground-breaking work of Thompson<br>and his co-workers (Thompson 1989*a*, b, 1990; Virgin 1987, 1 of the underlying phase space, as done in the ground-breaking work of Thompson<br>and his co-workers (Thompson 1989*a*, *b*, 1990; Virgin 1987, 1989), and more recently<br>by Spyrou (1996). A comprehensive review of the mechani and his co-workers (Thompson 1989 $a, b$ , 1990; Virgin 1987, 1989), and more recently<br>by Spyrou (1996). A comprehensive review of the mechanics of ship capsize using<br>global geometrical techniques is given by Thompson (1997) by Spyrou (1996). A comprehensive review of the mechanics of ship capsize using<br>global geometrical techniques is given by Thompson (1997). These simulation-based<br>methods offer insight, but do not yield analytical results t global geometrical techniques is given by Thompson (1997). These simulation-based<br>methods offer insight, but do not yield analytical results that allow for a priori<br>prediction of the effects of system parameters on the res methods offer insight, but do not yield analytical results that allow for *a priori* prediction of the effects of system parameters on the response of the ship to various types of excitation. In addition, all of the above prediction of the effects of system parameters on the response of the ship to various types of excitation. In addition, all of the above methods are applicable for deterministic wave excitation but have not been applied to ious types of excitation. In addition, all of the above methods are applicable for<br>deterministic wave excitation but have not been applied to extreme ship motions<br>under stochastic excitation. Some analytical methods have b  $\overline{0}$ deterministic wave excitation but have not been applied to extreme ship motions<br>under stochastic excitation. Some analytical methods have been used for analysing<br>roll dynamics in random seas, including the stochastic aver under stochastic excitation. Some analytical methods have been used for analysing roll dynamics in random seas, including the stochastic averaging method, introduced by Roberts (1982) and Roberts  $\&$  Dacunha (1985) for a roll dynamics in random seas, including the stochastic averaging method, introduced<br>by Roberts (1982) and Roberts & Dacunha (1985) for application to ship motions,<br>and its extension by Huang *et al.* (1994) to a system wit by Roberts (1982) a<br>and its extension by<br>restoring moment.<br>More recently con d its extension by Huang *et al.* (1994) to a system with a fifth-order polynomial<br>storing moment.<br>More recently, combined geometric/analytical methods have been applied to the<br>oblem of nonlinear ship dynamics. Instead of

problem of nonlinear ship dynamics. Instead of directly solving the nonlinear differential equations these methods emphasize the qualitative behaviour of the system More recently, combined geometric/analytical methods have been applied to the problem of nonlinear ship dynamics. Instead of directly solving the nonlinear differential equations, these methods emphasize the qualitative be problem of nonlinear ship dynamics. Instead of directly solving the nonlinear differential equations, these methods emphasize the qualitative behaviour of the system, and often allow for analytical estimates of important f ential equations, these methods emphasize the qualitative behaviour of the system,<br>and often allow for analytical estimates of important features of large-amplitude<br>responses. One of the more popular analytic techniques of **MATHEMATICAL,<br>PHYSICAL<br>& ENGINEERING<br>SCIENCES** and often allow for analytical estimates of important features of large-amplitude<br>responses. One of the more popular analytic techniques of this type is the Melnikov<br>method, which can predict parameter conditions under whi responses. One of the more popular analytic techniques of this type is the Melnikov method, which can predict parameter conditions under which capsize is possible. This approach, which is restricted to certain classes of s This approach, which is restricted to certain classes of system models, describes con-This approach, which is restricted to certain classes of system models, describes conditions under which solutions of the model equations can be transported from one region of the system phase space (e.g. safety) to anothe ditions under which solutions of the model equations can be transported from one<br>region of the system phase space (e.g. safety) to another (e.g. capsize). The works<br>of Wiggins (1990, 1992) describe the basic theory and an region of the system phase space (e.g. safety) to another (e.g. capsize). The works<br>of Wiggins (1990, 1992) describe the basic theory and an important general appli-<br>cation of Melnikov's method, called *chaotic transport t* of Wiggins (1990, 1992) describe the basic theory and an important general application of Melnikov's method, called *chaotic transport theory*. A significant result from this theory is that the rate at which solutions are from this theory is that the rate at which solutions are transported out of the safe<br>regions, called the *phase-space flux rate*, can be calculated from the Melnikov func-<br>tion. These ideas, originally developed for deter regions, called the *phase-space flux rate*, can be calculated from the Melnikov function. These ideas, originally developed for deterministic excitation, were applied to the capsize problem in harmonic waves by Falzarano  $\blacktriangleright$  regions, called the *phase-space flux rate*, can be calculated from the Melnikov function. These ideas, originally developed for deterministic excitation, were applied to<br>the capsize problem in harmonic waves by Falzarano *et al.* (1992). The method has<br>been extended to systems that have random excitation  $\alpha$  the capsize problem in harmonic waves by Falzarano *et al.* (1992). The method has  $\Box$  been extended to systems that have random excitation (Frey & Simiu 1993) and  $\Box$   $\Box$  applied to the problem of ship capsize in been extended to systems that have random excitation (Frey & Simiu 1993) and

In all the above methods, the coefficients used to model hydrodynamic loads are taken to be constants. However, in general, the hydrodynamic coefficients are fre-In all the above methods, the coefficients used to model hydrodynamic loads are taken to be constants. However, in general, the hydrodynamic coefficients are frequency dependent. For excitations of multiple or even infini taken to be constants. However, in general, the hydrodynamic coefficients are frequency dependent. For excitations of multiple or even infinite number of frequencies (e.g. random excitation), it is common to approximate th quency dependent. For excitations of multiple or even infinite number of frequencies<br>(e.g. random excitation), it is common to approximate the hydrodynamic effect by<br>simply using a coefficient computed at a particular freq (e.g. random excitation), it is common to approximate the hydrodynamic effect by simply using a coefficient computed at a particular frequency of interest. Another approach to this problem, and the one employed in the pres

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Capsize criteria with memory-dependent models 1763<br>frequency-dependent term as the response of an auxiliary dynamical system. Holappa<br>& Falzarano (1999) considered the extended state space associated with such a model. frequency-dependent term as the response of an auxiliary dynamical system. Holappa<br>& Falzarano (1999) considered the extended state space associated with such a model,<br>including nonlinear rolling effects and frequency-depe frequency-dependent term as the response of an auxiliary dynamical system. Holappa<br>& Falzarano (1999) considered the extended state space associated with such a model,<br>including nonlinear rolling effects and frequency-depe & Falzarano (1999) considered the extended state space associated with such a model, including nonlinear rolling effects and frequency-dependent coefficients. Using numerical integration, they found that acceptable results including nonlinear rolling effects and frequency-dependent coefficients. Using numerical integration, they found that acceptable results could be obtained for some cases<br>by using constant hydrodynamic coefficients evaluated at a frequency based upon<br>the zero crossing period of the sea spectrum. They did n by using constant h<br>the zero crossing per<br>leading to capsize.<br>In this paper, we c e zero crossing period of the sea spectrum. They did not consider extreme motions<br>In this paper, we consider the frequency dependence of the system's hydrodynamic<br>efficients and apply Melnikov's method to the extended dyna

leading to capsize.<br>In this paper, we consider the frequency dependence of the system's hydrodynamic<br>coefficients and apply Melnikov's method to the extended dynamical system that<br>includes the auxiliary system. The results In this paper, we consider the frequency dependence of the system's hydrodynamic<br>coefficients and apply Melnikov's method to the extended dynamical system that<br>includes the auxiliary system. The results are obtained for th coefficients and apply Melnikov's method to the extended dynamical system that<br>includes the auxiliary system. The results are obtained for the case of random sea<br>states. The paper is organized as follows. We begin with a r includes the auxiliary system. The results are obtained for the case of random sea<br>states. The paper is organized as follows. We begin with a review of the basic system<br>model, in which we incorporate memory-dependent radia states. The paper is organized as follows. We begin with a review of the basic system<br>model, in which we incorporate memory-dependent radiation effects in the nonlinear<br>differential equations, valid for large-amplitude rol model, in which we incorporate memory-dependent radiation effects in the nonlinear<br>differential equations, valid for large-amplitude roll dynamics. Attention is paid to the<br>relationships between the frequency and time-doma differential equations, valid for large-amplitude roll dynamics. Attention is paid to the relationships between the frequency and time-domain descriptions of these radiation terms. Next, the Melnikov function for this syst relationships between the frequency and time-domain descriptions of these radiation<br>terms. Next, the Melnikov function for this system model with random excitation<br>is derived and is described in terms of its statistical pr terms. Next, the Melnikov function for this system model with random excitation<br>is derived and is described in terms of its statistical properties. These results are<br>then linked to the likelihood of capsize through a measu is derived and is described in terms of its statistical properties. These results are then linked to the likelihood of capsize through a measure of the rate of phase-space transport. These results are used to study the eff then linked to the likelihood of capsize through a measure of the rate of phase-space transport. These results are used to study the effects of the damping model on the capsize criterion. Extensive simulations are used to transport. These results are used to study the effects of the damping model on the ome conclusions.<br>2. The model for rolling motion

# (*a*) *The basic model*

 $(a)$  *The basic model*<br>Ship rolling behaviour can be represented by the following SDOF equation (see, for example. Jiang *et al.* 1994): Ship rolling behaviour can be<br>example, Jiang *et al.* 1994):

$$
(I_{44} + A_{44}(\omega))\ddot{\phi} + B_{44}(\omega)\dot{\phi} + B_{44q}(\omega)\dot{\phi}|\dot{\phi}| + \Delta(C_0 + C_1\phi + C_3\phi^3 + \cdots) = F(\tau),
$$
\n(2.1)

*IATHEMATICAL,<br>'HYSICAL<br>k ENGINEERING<br>CIENCES* where  $\phi$  is the roll angle in an absolute reference frame,  $\phi$  = d/d $\tau$ ,  $I_{44}$  is the moment of inertia of the dry vessel with respect to an axis along an assumed roll where  $\phi$  is the roll angle in an absolute reference frame,  $\dot{\left( \right)} = d/d\tau$ ,  $I_{44}$  is the moment of inertia of the dry vessel with respect to an axis along an assumed roll centre.  $F(\tau)$  is the moment due to incident w where  $\phi$  is the roll angle in an absolute reference frame, () =  $d/d\tau$ ,  $I_{44}$  is the moment of inertia of the dry vessel with respect to an axis along an assumed roll centre,  $F(\tau)$  is the moment due to incident waves moment of inertia of the dry vessel with respect to an axis along an assumed roll<br>centre,  $F(\tau)$  is the moment due to incident waves with respect to the same axis,<br> $A_{44}(\omega)$  is the added mass coefficient,  $B_{44}(\omega)$  and

centre,  $F(\tau)$  is the moment due to incident waves with respect to the same axis,<br> $A_{44}(\omega)$  is the added mass coefficient,  $B_{44}(\omega)$  and  $B_{44q}(\omega)$  are linear and quadratic<br>damping coefficients, respectively,  $\Delta$  is t  $A_{44}(\omega)$  is the added mass coefficient,  $B_{44}(\omega)$  and  $B_{44q}(\omega)$  are linear and quadratic damping coefficients, respectively,  $\Delta$  is the displacement of the vessel,  $C_1$  and  $C_3$  are linear and nonlinear coefficien damping coefficients, respectively,  $\Delta$  is the displacement of the vessel,  $C_1$  and  $C_3$  are<br>linear and nonlinear coefficients of the restoring arm, and  $C_0$  the bias moment which<br>can arise due to wind, cargo, vessel linear and nonlinear coefficients of the restoring arm, and  $C_0$  the bias moment which<br>can arise due to wind, cargo, vessel damage or the pull of a fishing net. (Note that<br>the nonlinear restoring moment is assumed to be can arise due to wind, cargo, vessel damage or the nonlinear restoring moment is assumed to be derived from a curve fit or a series expansion.)<br> $A_{AA}(\omega)$  and  $B_{AA}(\omega)$  are frequency dependent b e nonlinear restoring moment is assumed to be valid for large angles, that is, it is<br>rived from a curve fit or a series expansion.)<br> $A_{44}(\omega)$  and  $B_{44}(\omega)$  are frequency dependent because of the presence of the free<br>rfa

derived from a curve fit or a series expansion.)<br> $A_{44}(\omega)$  and  $B_{44}(\omega)$  are frequency dependent because of the presence of the free<br>surface. For a single sinusoidal excitation  $F(\tau)$  and sinusoidal response, they take<br>  $A_{44}(\omega)$  and  $B_{44}(\omega)$  are frequency dependent because of the presence of the free<br>surface. For a single sinusoidal excitation  $F(\tau)$  and sinusoidal response, they take<br>values corresponding to the excitation frequency. surface. For a single sinusoidal excitation  $F(\tau)$  and sinusoidal response, they take<br>values corresponding to the excitation frequency. Only in this case can (2.1) exactly<br>describe the linear ship roll hydrodynamics (Ogil values corresponding to the excitation frequency. Only in this case can (2.1) exactly<br>describe the linear ship roll hydrodynamics (Ogilvie 1964). If the excitation is not<br>purely sinusoidal, e.g. for random excitation,  $A_{$ describe the linear ship roll hydrodynamics (Ogilvie 1964). If the excitation is not purely sinusoidal, e.g. for random excitation,  $A_{44}(\omega)$  and  $B_{44}(\omega)$  are no longer constant and (2.1) is an approximation (see Takag purely sinusoidal, e.g. for random excitation,  $A_{44}(\omega)$  and  $B_{44}(\omega)$  are no longer constant and (2.1) is an approximation (see Takagi *et al.* 1984; Jiang *et al.* 1996).<br>For excitation represented by narrow-banded sp stant and (2.1) is an approximation (see Takagi *et al.* 1984; Jiang *et al.* 1996).<br>For excitation represented by narrow-banded spectra,  $A_{44}(\omega)$  and  $B_{44}(\omega)$  may be evaluated at the dominant excitation frequency, sa For excitation represented by narrow-banded spectra,  $A_{44}(\omega)$  and  $B_{44}(\omega)$  may be evaluated at the dominant excitation frequency, say,  $\omega_z$  (see, for example, Esparza & Falzarano 1993; Holappa & Falzarano 1999). For evaluated at the dominant excitation frequency, say,  $\omega_z$  (see, for & Falzarano 1993; Holappa & Falzarano 1999). For excitations spectra, the values at the roll natural frequency may be better. *Phil. Trans. R. Soc. Lond.* A (2000)

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### (*b*) *A frequency-dependent model for random excitation*

To improve this frequency-domain description, a modified model is needed—one To improve this frequency-domain description, a modified model is needed—one<br>that can be represented in the time domain. In linear hydrodynamics, the ship hydro-<br>dynamic forces can be viewed as the output of a linear syst To improve this frequency-domain description, a modified model is needed—one<br>that can be represented in the time domain. In linear hydrodynamics, the ship hydro-<br>dynamic forces can be viewed as the output of a linear syst dynamic forces can be viewed as the output of a linear system with the wave elevation  $(\zeta(\tau))$  and roll motion  $(\phi(\tau), \dot{\phi}(\tau))$  and  $\ddot{\phi}(\tau)$  as its inputs. The output corredynamic forces can be viewed as the output of a linear system with the wave elevation  $(\zeta(\tau))$  and roll motion  $(\phi(\tau), \dot{\phi}(\tau))$  and  $\ddot{\phi}(\tau)$ ) as its inputs. The output corresponding to  $\zeta(\tau)$  is the so-called external tion  $(\zeta(\tau))$  and roll motion  $(\phi(\tau), \phi(\tau))$  as<br>sponding to  $\zeta(\tau)$  is the so-called external<br> $\zeta(\tau)$  in the frequency domain as follows: as follows:<br>  $f_f^+(\omega) = |F_{roll}(\omega)|^2 S_\zeta^+(\omega)$ 

$$
S_{\rm f}^+(\omega) = |F_{\rm roll}(\omega)|^2 S_{\zeta}^+(\omega),\tag{2.2}
$$

 $S_{\rm f}^{+}(\omega) = |F_{\rm roll}(\omega)|^2 S_{\zeta}^{+}(\omega),$ <br>where  $F_{\rm roll}(\omega)$  is the roll moment amplitude per unit wave height,  $S_{\zeta}^{+}(\omega)$ <br>are the wave elevation spectrum and the wave-induced roll moment spect:  $+$ (...)  $(2.2)$ <br>  $\chi^+_{\zeta}(\omega)$  and  $S^+_{\text{f}}(\omega)$ <br>
nectrum, respectively  $\gamma_{\rm f}^{\prime +}(\omega)$ where  $F_{\text{roll}}(\omega)$  is the roll moment amplitude per unit wave height,  $S_{\zeta}^{+}(\omega)$  and  $S_{\text{f}}^{+}(\omega)$  are the wave-elevation spectrum and the wave-induced roll moment spectrum, respectively.  $F_{\text{coll}}(\omega)$  depends on fre where  $F_{\text{roll}}(\omega)$  is the roll moment amplitude per unit wave height,  $S_{\zeta}^{+}(\omega)$  and  $S_{\zeta}^{+}(\omega)$  are the wave elevation spectrum and the wave-induced roll moment spectrum, respectively.  $F_{\text{roll}}(\omega)$  depends on freq are the wave elevation spectrum and the wave-induced roll moment spectrum, respectively.  $F_{roll}(\omega)$  depends on frequency as well as ship geometry. The wave elevation  $\zeta(\tau)$  is usually assumed to a stationary ergodic Gaus tively.  $F_{roll}(\omega)$  depends on frequency as well as ship ge  $\zeta(\tau)$  is usually assumed to a stationary ergodic Gauss accepted model is the ISSC two-parameter spectrum, accepted model is the ISSC two-parameter spectrum,

$$
S_{\zeta}^{+}(\omega) = 0.11 H_{\zeta}^{2} \frac{\omega_{z}^{4}}{\omega^{5}} \exp\left(-0.44\left(\frac{\omega_{z}}{\omega}\right)^{4}\right),
$$
 (2.3)  
where  $H_{\zeta}$  is the significant wave height and  $\omega_{z}$  is the characteristic wave frequency.  
The hydrodynamic moment associated with  $\phi(\tau)$  is the 'hydrostatic' restoring

The hydrodynamic moment associated with  $\phi(\tau)$  is the characteristic wave frequency.<br>The hydrodynamic moment associated with  $\phi(\tau)$  is the 'hydrostatic' restoring<br>nent that is  $\Delta(C_0 + C_1\phi + C_2\phi^3 + \cdots)$  in (2.1) the cha where  $H_s$  is the significant wave height a<br>The hydrodynamic moment associated<br>moment, that is,  $\Delta(C_0 + C_1\phi + C_3\phi^3 +$ <br>depend only upon ship geometry. The hy in and  $\omega_z$  is the characteristic wave frequency.<br>ated with  $\phi(\tau)$  is the 'hydrostatic' restoring<br> $3 + \cdots$ ) in (2.1), the characteristics of which<br>e hydrodynamic reaction moment due to  $\phi(\tau)$ The hydrodynamic moment associated with  $\phi(\tau)$  is the 'hydrostatic' restoring<br>moment, that is,  $\Delta(C_0 + C_1\phi + C_3\phi^3 + \cdots)$  in (2.1), the characteristics of which<br>depend only upon ship geometry. The hydrodynamic reaction m moment, that is,  $\Delta(C_0 + C_1\phi + C_3\phi^3 + \cdots)$  in (2.1), the characteristics of which<br>depend only upon ship geometry. The hydrodynamic reaction moment due to  $\dot{\phi}(\tau)$ <br>and  $\ddot{\phi}(\tau)$  is conceptually identified with the rad depend only upon ship geometry. The hydrodynamic reaction moment due to  $\phi(\tau)$  and  $\ddot{\phi}(\tau)$  is conceptually identified with the radiation, i.e. forced motion, problem.<br>Mathematically, the linear radiation force can be and  $\phi(\tau)$  is conceptually identified with the radiation, i.e. forced motion, problem.<br>Mathematically, the linear radiation force can be taken as the sum of a succession of<br>impulse responses. Following Ogilvie (1964), th is

$$
(I_{44} + A_{44}(\infty))\ddot{\phi} + B_{44}(\infty)\dot{\phi} + \int_0^{\tau} K(\tau - u)\dot{\phi}(u) du + B_{44q}(\omega)\dot{\phi}|\dot{\phi}| + \Delta(C_0 + C_1\phi + C_3\phi^3 + \cdots) = F(\tau),
$$
 (2.4)

where  $A_{44}(\infty)$  is the hydrodynamic added mass coefficient evaluated at the infinite where  $A_{44}(\infty)$  is the hydrodynamic added mass coefficient evaluated at the infinite<br>frequency limit and  $K(\tau)$  is the hydrodynamic rolling moment due to impulse roll<br>velocity. The integral of the  $K(\tau)$  term is usually where  $A_{44}(\infty)$  is the hydrodynamic added mass coefficient evaluated at the infinite<br>frequency limit and  $K(\tau)$  is the hydrodynamic rolling moment due to impulse roll<br>velocity. The integral of the  $K(\tau)$  term is usually frequency limit and  $K(\tau)$  is the hydrodynamic rolling moment due to impulse roll<br>velocity. The integral of the  $K(\tau)$  term is usually called the memory function because<br>it represents how roll-radiation moments depend on velocity. The integral of the  $K(\tau)$  term is usually called the memory function because it represents how roll-radiation moments depend on the history of rolling velocity.<br>Modelled in this way, the hydrodynamic forces (moments) change instantaneously with  $\phi(\tau)$  and  $\phi(\tau)$ , but the influence from  $\phi(\tau)$  i Modelled in this way, the hydrodynamic forces (moments) change instantaneously<br>with  $\phi(\tau)$  and  $\phi(\tau)$ , but the influence from  $\dot{\phi}(\tau)$  is cumulative and will be present<br>for sometime before it dies out.<br>The time- and f with  $\phi(\tau)$  and  $\phi(\tau)$ , but the influence from  $\phi(\tau)$  is cumulative and will be present

for sometime before it dies out.<br>The time- and frequency-domain descriptions of rolling motion are connected the following relation (Ogilvie 1964; Takagi *et al.* 1984):

$$
\text{gph the following relation (Ogilvie 1964; Takagi } et al. 1984):
$$
\n
$$
K(\tau) = -\frac{2}{\pi} \int_0^\infty \omega(A_{44}(\omega) - A_{44}(\infty)) \sin(\omega \tau) \, \mathrm{d}\omega \tag{2.5}
$$

$$
= -\frac{1}{\pi} \int_0^{\infty} \omega(A_{44}(\omega) - A_{44}(\infty)) \sin(\omega \tau) d\omega
$$
\n
$$
= \frac{2}{\pi} \int_0^{\infty} (B_{44}(\omega) - B_{44}(\infty)) \cos(\omega \tau) d\omega.
$$
\n(2.6)

 $=\frac{1}{\pi}\int_0^{\pi} (B_{44}(\omega) - B_{44}(\infty)) \cos(\omega \tau) d\omega.$  (2.6)<br>To obtain a time-domain description like (2.4), the hydrodynamic problem may be<br>solved in the frequency domain for a range of frequencies. This may be done by apply-To obtain a time-domain description like  $(2.4)$ , the hydrodynamic problem may be<br>solved in the frequency domain for a range of frequencies. This may be done by apply-<br>ing commercially available linear hydrodynamic progra To obtain a time-domain description like (2.4), the hydrodynamic problem may be solved in the frequency domain for a range of frequencies. This may be done by applying commercially available linear hydrodynamic programs, e *Phil. Trans. R. Soc. Lond.* A (2000)

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Figure 1. The added mass  $(A_{44}(\omega)-A_{44}(\infty))$  and linear damping  $(B_{44}(\omega)-B_{44}(\infty))$  coefficients Figure 1. The added mass  $(A_{44}(\omega) - A_{44}(\infty))$  and linear damping  $(B_{44}(\omega) - B_{44}(\infty))$  coef<br>obtained from SHIPMO for *Patti-B*. The units are N m<sup>-1</sup> s<sup>-2</sup> for  $A_{44}$  and N m<sup>-1</sup> s<sup>-1</sup> for<br> $A_{44}(\infty) = 1.441 \times 10^6$  and  $B$ Figure 1. The added mass  $(A_{44}(\omega) - A_{44}(\infty))$  and linear damping  $(B_{44}(\omega) - B_{44}(\infty))$  coefficients obtained from SHIPMO for *Patti-B*. The units are N m<sup>-1</sup> s<sup>-2</sup> for  $A_{44}$  and N m<sup>-1</sup> s<sup>-1</sup> for  $B_{44}$ .<br> $A_{44}(\infty) = 1.4$ 

& Troesch 1990). The results for *Patti-B*, a fishing vessel, are shown in figure 1 and the corresponding  $K(t)$  obtained from (2.6) is shown in figure 2. It should be noted & Troesch 1990). The results for *Patti-B*, a fishing vessel, are shown in figure 1 and<br>the corresponding  $K(t)$ , obtained from (2.6), is shown in figure 2. It should be noted<br>that  $A_{\mathcal{U}}(\omega)$  and  $B_{\mathcal{U}}(\omega)$  are both & Troesch 1990). The results for *Patti-B*, a fishing vessel, are shown in figure 1 and<br>the corresponding  $K(t)$ , obtained from (2.6), is shown in figure 2. It should be noted<br>that  $A_{44}(\omega)$  and  $B_{44}(\omega)$  are both even f the corresponding  $K(t)$ , obtained from (2.6), is shown in figure 2. It should be noted<br>that  $A_{44}(\omega)$  and  $B_{44}(\omega)$  are both even functions of  $\omega$ . Therefore,  $A_{44}(\omega) - A_{44}(\infty)$ <br>and  $B_{44}(\omega) - B_{44}(\infty)$  are also even that  $A_{44}(\omega)$  and  $B_{44}(\omega)$  are both even functions of  $\omega$ . Therefore,  $A_{44}(\omega) - A_{44}(\infty)$ <br>and  $B_{44}(\omega) - B_{44}(\infty)$  are also even and the negative frequency domain is omit-<br>ted in figure 1. The Fourier transform of m and  $B_{44}(\omega) - B_{44}(\infty)$  are also even and the negative frequency domain is omit-<br>ted in figure 1. The Fourier transform of memory function  $K(\tau)$  is related to the<br>hydrodynamic coefficients as follows (Takagi *et al.* 19

$$
\mathcal{K}(\omega) = \frac{1}{2\pi} [B_{44}(\omega) - B_{44}(\infty) + i\omega (A_{44}(\omega) - A_{44}(\infty))],
$$
\n(2.7)

where the  $1/2\pi$  may not be needed, depending on the definition of the Fourier transform. nere the  $1/2\pi$  may not be needed, depending on the definition of the Fourier trans-<br>The quadratic damping coefficient  $B_{44q}(\omega)$  in (2.1) and (2.4) is found by experi-<br>ental tests. While it may have some small frequenc

form.<br>The quadratic damping coefficient  $B_{44q}(\omega)$  in (2.1) and (2.4) is found by experi-<br>mental tests. While it may have some small frequency dependence (Himeno 1981),<br>in this work it is treated as constant. The linear The quadratic damping coefficient  $B_{44q}(\omega)$  in (2.1) and (2.4) is found by experi-<br>mental tests. While it may have some small frequency dependence (Himeno 1981),<br>in this work it is treated as constant. The linear roll d frequency dependence (Himeno 1981), in this work it is treated as constant. The linear roll damping goes to zero as the frequency goes to infinity and roll velocity is bounded, thereby eliminating the term in this work it is treated as constant. The linear roll damping goes to zero as the frequency goes to infinity and roll velocity is bounded, thereby eliminating the term  $B_{44}(\infty)\dot{\phi}$  in (2.4). Note also that the added frequency goes to infinity and roll velocity is bounded, thereby eliminating the term  $B_{44}(\infty)\dot{\phi}$  in (2.4). Note also that the added mass term tends to a non-zero constant as the frequency goes to infinity (figure 1 s as the frequency goes to infinity (figure 1 shows  $A_{44}(\omega) - A_{44}(\infty)$  instead of  $A_{44}(\omega)$ ).<br>From casuality, it is understood that the impulse response function  $K(\tau)$  is zero for  $\tau < 0$ . Since  $K(\tau < 0) = 0$  and  $\dot{\phi}(\$ From casuality, it is understood that the impulse response function  $K(\tau)$  is zero for

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Figure 2. The impulse response  $K(\tau)$  function calculated from the inverse cosine transform of  $B_{44}(\omega) - B_{44}(\infty)$ . The unit is  $N m^{-1} B_{44}(\infty) = 0$  $B_{44}(\omega) - B_{44}(\infty)$ . The unit is N m<sup>-1</sup>.  $B_{44}(\infty) = 0$ . from the inverse<br> $B_{44}(\infty) = 0.$ 

expanded to  $(-\infty, \infty)$  and  $(2.4)$  is rewritten as

$$
(I_{44} + A_{44}(\infty))\ddot{\phi} + \int_{-\infty}^{\infty} K(\tau - u)\dot{\phi}(u) du
$$
  
+ 
$$
B_{44q}(\omega)\dot{\phi}|\dot{\phi}| + \Delta(C_0 + C_1\phi + C_3\phi^3 + \cdots) = F(\tau).
$$
 (2.8)

To distinguish this from the constant coefficient differential equation, equation  $(2.8)$ is called the integro-differential equation. For the purpose of comparison with previ-To distinguish this from the constant coefficient differential equation, equation (2.8) is called the integro-differential equation. For the purpose of comparison with previously published results (Hsieh *et al.* 1994; Ji is called the integro-differential equation. For the purpose of comparison with previ-<br>ously published results (Hsieh *et al.* 1994; Jiang *et al.* 1996), we simplify the restoring<br>moment by taking  $C_i = 0$  for  $i > 3$ . How ously published results (Hsieh *et al.* 1994; Jiang *et al.* 1996), we simpli<br>moment by taking  $C_i = 0$  for  $i > 3$ . However, the method introduced<br>handle any form of angle-dependent hydrostatic restoring moment. handle any form of angle-dependent hydrostatic restoring moment.<br>(*c*) *Scaling the equations of motion* 

In large-amplitude roll, nonlinear effects in the restoring moment can easily dominate the behaviour. The method used in this work is a global perturbation method In large-amplitude roll, nonlinear effects in the restoring moment can easily dominate the behaviour. The method used in this work is a global perturbation method that allows one to predict certain features of such nonline inate the behaviour. The method used in this work is a global perturbation method<br>that allows one to predict certain features of such nonlinear behaviour. It is based<br>on determining the effects that relatively small dampin that allows one to predict certain features of such nonlinear behaviour. It is based<br>on determining the effects that relatively small damping and excitation have on the<br>overall nonlinear system behaviour. In fact, as shown on determining the effects that relatively small damping and excitation have on the overall nonlinear system behaviour. In fact, as shown below, the terms in the roll equation of motion scale exactly as needed for applicat erall nonlinear system behaviour. In fact, as shown below, the terms in the roll<br>uation of motion scale exactly as needed for application of this method.<br>To compare the relative orders of the various terms in the roll equa

equation of motion scale exactly as needed for application of this method.<br>To compare the relative orders of the various terms in the roll equation, we rewrite<br>the usual roll differential equation  $(2.1)$  and the history-To compare the relative orders of the various terms:<br>the usual roll differential equation  $(2.1)$  and the history<br>equation  $(2.8)$  in non-dimensional forms, as follows: characterization (2.1) and the instead operator.<br>  $\ddot{x}(t) + \epsilon \delta_1 \dot{x}(t) + \epsilon \delta_2 \dot{x}(t) |\dot{x}(t)| + \delta + x(t) - \alpha x^3(t)$ (t) =  $\epsilon f(t)$  (2.9)

$$
\ddot{x}(t) + \epsilon \delta_1 \dot{x}(t) + \epsilon \delta_2 \dot{x}(t) |\dot{x}(t)| + \delta + x(t) - \alpha x^3(t) = \epsilon f(t) \qquad (2.9)
$$

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$$
\ddot{x}(t) + \epsilon \int_{-\infty}^{\infty} \delta_{\rm m}(t - u)\dot{x}(u) \, \mathrm{d}u + \epsilon \delta_2 \dot{x}(t) |\dot{x}(t)| + \delta + x(t) - \alpha x^3(t) = \epsilon f(t), \quad (2.10)
$$

where

$$
x = \phi, \qquad t = \omega_{\rm n}\tau, \qquad (\ ) = \frac{\mathrm{d}}{\mathrm{d}t},
$$

$$
\omega_{\rm n} = \sqrt{\frac{C_1\Delta}{I_{44} + A_{44}}}, \qquad \Omega = \frac{\omega}{\omega_{\rm n}},
$$

$$
\epsilon \delta_1 = \frac{B_{44}\omega_{\rm n}}{C_1\Delta}, \qquad \epsilon \delta_{\rm m}(t) = \frac{K(t)\omega_{\rm n}^2}{C_1\Delta} = \frac{K(t)}{I_{44} + A_{44}}, \qquad \epsilon \delta_2 = \frac{B_{44\rm q}}{I_{44} + A_{44}},
$$

$$
\delta = \frac{C_0}{C_1}, \qquad \alpha = \frac{-C_3}{C_1}, \qquad \epsilon f(t) = \frac{F(\tau)}{C_1\Delta}.
$$

 $\delta = \frac{\epsilon_0}{C_1}, \qquad \alpha = \frac{\epsilon_3}{C_1}, \qquad \epsilon f(t) = \frac{f(t)}{C_1}$ .<br>Note that  $A_{44}$  in (2.9) can be taken as  $A_{44} = A_{44}(\omega_n)$  or  $A_{44} = A_{44}(\omega_z)$ , and that  $A_{44} = A_{44}(\infty)$  for (2.10). The quadratic damping coefficient  $B_{44}(\omega)$ Note that  $A_{44}$  in (2.9) can be taken as  $A_{44} = A_{44}(\omega_n)$  or  $A_{44} = A_{44}(\omega_z)$ , and<br>that  $A_{44} = A_{44}(\infty)$  for (2.10). The quadratic damping coefficient  $B_{44q}(\omega)$  is the<br>same for both cases. The various coefficients Note that  $A_{44}$  in (2.9) can be taken as  $A_{44} = A_{44}(\omega_n)$  or  $A_{44} = A_{44}(\omega_z)$ , and that  $A_{44} = A_{44}(\infty)$  for (2.10). The quadratic damping coefficient  $B_{44q}(\omega)$  is the same for both cases. The various coefficients that  $A_{44} = A_{44}(\infty)$  for (2.10). The quadratic damping coefficient  $B_{44q}(\omega)$  is the same for both cases. The various coefficients for the *Patti-B* are listed in table 1.<br>Compared with the added mass and linear restor same for both cases. The various coefficients for the *Patti-B* are listed in table 1.<br>Compared with the added mass and linear restoring moment terms, which are of order 1, the nonlinear restoring coefficient  $\alpha$  is 3.14 Compared with the added mass and linear restoring moment terms, which are of order 1, the nonlinear restoring coefficient  $\alpha$  is 3.14, the external excitation amplitude per unit wave height at a typical frequency is  $F_{\$ order 1, the nonlinear restoring coefficient  $\alpha$  is 3.14, the external excitation amplitude per unit wave height at a typical frequency is  $F_{roll}(\omega = 0.7) = 0.006$  and the quadratic damping coefficient  $\epsilon \delta_2$  is 0.067. Th tude per unit wave height at a typical frequency is  $F_{roll}(\omega = 0.7) = 0.006$  and<br>the quadratic damping coefficient  $\epsilon \delta_2$  is 0.067. The maximum value of the memory<br>function  $\epsilon \delta_m(t)|_{\text{maximum}}$  is 0.050 for the integro-differen the quadratic damping coefficient  $\epsilon \delta_2$  is 0.067. The maximum value of the memory function  $\epsilon \delta_{\rm m}(t)|_{\rm maximum}$  is 0.050 for the integro-differential system (equation (2.10)), while the constant linear damping coefficien function  $\epsilon \delta_{\rm m}(t)|_{\rm maximum}$  is 0.050 for the integro-differential system (equation (2.10)), while the constant linear damping coefficient  $\epsilon \delta_1$  is 0.004 for the usual differential equation model given in (2.9). Therefo while the constant linear damping coefficient  $\epsilon \delta_1$  equation model given in (2.9). Therefore, the use damping and wave excitation terms is justified.

## % and wave excitation terms is justified.<br>
3. A capsize criterion for models with memory-dependent<br>
bydrodynamic forces n for models with mem<br>hydrodynamic forces (*a*) *The system phase space and its safe basin*

(a) The system phase space and its safe basin<br>It is convenient to express  $(2.10)$  in first-order form, as follows:

It is convenient to express (2.10) in first-order form, as follows:  
\n
$$
\dot{x}(t) = y(t),
$$
\n(3.1)

$$
\dot{x}(t) = y(t),
$$
\n(3.1)  
\n
$$
\dot{y}(t) = -\delta - x(t) + \alpha x^3(t) + \epsilon \left( \int_{-\infty}^{\infty} \delta_m(t - u) y(u) \, du - \delta_2 y(t) |y(t)| + f(t) \right).
$$
\n(3.2)

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1768 C. Jiang, A. W. Troesch and  $\check{S}$ . W. Shaw<br>Note that (3.1) and (3.2) represent an integrable Hamiltonian system with some<br>small perturbations terms (Wiggins 1990). The unperturbed system ( $\epsilon = 0$ ) Note that (3.1) and (3.2) represent an integrable Hamiltonian system with small perturbations terms (Wiggins 1990). The unperturbed system  $(\epsilon = 0)$ small perturbations terms (Wiggins 1990). The unperturbed system ( $\epsilon = 0$ )

$$
\dot{x}(t) = y(t),\tag{3.3}
$$

$$
\dot{x}(t) = y(t),
$$
\n(3.3)\n  
\n
$$
\dot{y}(t) = -\delta - x(t) + \alpha x^3(t)
$$
\n(3.4)

 $\dot{y}(t) = -\delta - x(t) + \alpha x^3(t)$ <br>is a conservative two-dimensional system with solutions as shown in the  $(x, y)$  phase<br>planes given in figures 3 and 4. Except for some special cases (Falzarano *et al.* 1992: is a conservative two-dimensional system with solutions as shown in the  $(x, y)$  phase<br>planes given in figures 3 and 4. Except for some special cases (Falzarano *et al.* 1992;<br>Huang *et al.* 1994), equations (3.3) and (3.4) is a conservative two-dimensional system with solutions as shown in the  $(x, y)$  phase<br>planes given in figures 3 and 4. Except for some special cases (Falzarano *et al.* 1992;<br>Huang *et al.* 1994), equations (3.3) and (3.4) planes given in figures 3 and 4. Except for some special cases (Falzarano *et al.* 1992;<br>Huang *et al.* 1994), equations (3.3) and (3.4) must be solved numerically. For the<br>symmetric unbiased system ( $\delta = 0$ ) shown in fig Huang *et al.* 1994), equations (3.3) and (3.4) must be solved numerically. For the symmetric unbiased system ( $\delta = 0$ ) shown in figure 3, the two saddle points, corresponding to the angles of vanishing stability, are con symmetric unbiased system ( $\delta = 0$ ) shown in figure 3, the two saddle points, corresponding to the angles of vanishing stability, are connected by two symmetrical orbits known as heteroclinic orbits (shown as dashed lines  $y_0^+(t)$  and  $y_0^-(t)$ responding to the angles of vanishing stability, are connected by two symmetrical<br>orbits known as heteroclinic orbits (shown as dashed lines). We use  $y_0^+(t)$  and  $y_0^-(t)$ <br>to denote the ordinates of the upper branch and orbits known as heteroclinic orbits (shown as dashed lines). We use  $y_0^+(t)$  and  $y_0^-(t)$  to denote the ordinates of the upper branch and the lower branch, respectively, of these orbits in the phase plane as functions o to denote the ordinates of the upper branch and the lower branch, respectively, of these orbits in the phase plane as functions of time. Motions taking place inside the region enclosed by the heteroclinic orbits are bounde these orbits in the phase plane as functions of time. Motions taking place inside the region enclosed by the heteroclinic orbits are bounded and safe, in terms of capsize. This region is called the *safe basin* of the unpe region enclosed by the heteroclinic orbits are bounded and safe, in terms of capsize.<br>This region is called the *safe basin* of the unperturbed system, and the heteroclinic<br>orbits are the *basin boundaries*. Motions outsid This region is called the *safe basin* of the unperturbed system, and the heteroclinic orbits are the *basin boundaries*. Motions outside of this region lead to capsize. In the asymmetric biased system shown in figure 4, orbits are the *basin boundaries*. Motions outside of this region lead to capsize. In the asymmetric biased system shown in figure 4, the two saddle points are not connected but the left saddle point is connected to itself asymmetric biased system shown in figure 4, the two saddle points are not connected<br>but the left saddle point is connected to itself by a homoclinic orbit, which forms the<br>boundary of a smaller safe basin. The ordinate of but the left saddle point is connected to itself by a homoclinic orbit, which for boundary of a smaller safe basin. The ordinate of the homoclinic orbit is den  $y_\delta(t)$ . The area of the safe basin can be obtained by integr

sin can be obtained by integration as follows:  
\n
$$
A_0 = 2 \int_{x_1}^{x_2} y_0^+(t) dx_0(t)
$$
\n
$$
= 2 \int_{-\infty}^{\infty} [y_0^+(t)]^2 dt
$$
\n(3.5)

 $= 2 \int_{-\infty} [y_0'(t)]^2 dt$  (3.5)<br>for the unbiased system, where  $x_1$  and  $x_2$  are the extrema of the roll angle of the<br>safe basin and for the unbiased sysafe basin, and

$$
A_{\delta} = 2 \int_0^{\infty} y_{\delta}^2(t) dt
$$
 (3.6)

 $A_{\delta} = 2 \int_{0}^{t} y_{\delta}^{*}(t) dt$  (3.6)<br>for the biased system, where  $t = 0$  is taken to be the point at which  $y_{\delta} = 0$  away<br>from the saddle point. for the biased system, v<br>from the saddle point.<br>Heteroclinic and hom the biased system, where  $t = 0$  is taken to be the point at which  $y_{\delta} = 0$  away<br>m the saddle point.<br>Heteroclinic and homoclinic orbits are also known as separatrices, because they<br>parate bounded and unbounded motions. T

from the saddle point.<br>Heteroclinic and homoclinic orbits are also known as separatrices, because they<br>separate bounded and unbounded motions. The generalization of these boundaries<br>to the damped forced ship motion is the Heteroclinic and homoclinic orbits are also known as separatrices, because they separate bounded and unbounded motions. The generalization of these boundaries to the damped forced ship motion is the key in analysing the gl separate bounded and unbounded motions. The generalization of these boundaries<br>to the damped forced ship motion is the key in analysing the global stability of this<br>dynamical system. Figure 5 shows the ordinates of the sep to the damped forced ship motion is the key in analysing the global stability of this dynamical system. Figure 5 shows the ordinates of the separatrices for different heel angles, that is, for different levels of bias. It dynamical system. Figure 5 shows the ordinates of the separatrices for different angles, that is, for different levels of bias. It is noted that, by proper choice of starting time,  $y_0^+(t)$  is an even function of  $t$  and gles, that is, for different levels of bias. It is noted that, by proper choice of the arting time,  $y_0^+(t)$  is an even function of  $t$  and  $y_\delta(t)$  is an odd function of  $t$ .<br>In the unperturbed system, i.e. the unforced

starting time,  $y_0^+(t)$  is an even function of t and  $y_\delta(t)$  is an odd function of t.<br>In the unperturbed system, i.e. the unforced and undamped system, the safe<br>motions stay bounded forever. Under very small harmonic exc In the unperturbed system, i.e. the unforced and undamped system, the safe motions stay bounded forever. Under very small harmonic excitation and small damping, the motion generally remains periodic and bounded. As the exc motions stay bounded forever. Under very small harmonic excitation and small damping, the motion generally remains periodic and bounded. As the excitation levels are increased, the motion becomes more complicated, even cha ing, the motion generally remains periodic and bounded. As the excitation levels are<br>increased, the motion becomes more complicated, even chaotic. Beyond a critical<br>level of excitation, portions of the safe basin that are increased, the motion becomes more complicated, even chaotic. Beyond a critical<br>level of excitation, portions of the safe basin that are close to the unperturbed bound-<br>ary may be transported out and become unbounded, res level of excitation, portions of the safe basin that are close to the unperturbed bound-<br>ary may be transported out and become unbounded, resulting in capsize (Thomp-<br>son 1989*a*,*b*; Virgin 1987, 1989; Falzarano *et al*. ary may be transported out and become unbounded, resulting in capsize (Thomp-

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*Capsize cr[iteria with memory-depen](http://rsta.royalsocietypublishing.org/)dent models* <sup>1769</sup> Downloaded from rsta.royalsocietypublishing.org



Figure 3. Phase plane for  $\delta = 0$ . Heteroclinic separatrix,  $\mathcal{W}_0$ , is shown as dashed lines.<br>The two saddles are  $(-0.564, 0.0)$  and  $(+0.564, 0.0)$ blane for  $\delta = 0$ . Heteroclinic separatrix,  $\mathcal{W}_0$ , is shown at The two saddles are  $(-0.564, 0.0)$  and  $(+0.564, 0.0)$ .



**MATHEMATICAL,<br>PHYSICAL<br>& ENGINEERING<br>SCIENCES** Figure 4. Phase plane for  $\delta = 0.070$ , equivalent to  $4^{\circ}$  heel angle. Homoclinic separatrix,  $\mathcal{W}_{\delta}$ , is shown as dashed lines. The two saddles are  $(-0.526, 0.0)$  and  $(+0.597, 0.0)$ Phase plane for  $\delta = 0.070$ , equivalent to 4° heel angle. Homoclinic separatrix shown as dashed lines. The two saddles are  $(-0.526, 0.0)$  and  $(+0.597, 0.0)$ .

shown as dashed lines. The two saddles are  $(-0.526, 0.0)$  and  $(+0.597, 0.0)$ .<br>but starts to become significant at a particular level of excitation amplitude (Hsieh<br>et al. 1994). In this case, the rate of phase-space trans but starts to become significant at a particular level of excitation amplitude (Hsieh *et al.* 1994). In this case, the rate of phase-space transport can be quantified by a well-defined Melnikov function (Hsieh *et al.* 19 but starts to become significant at a particular level *et al.* 1994). In this case, the rate of phase-space tr well-defined Melnikov function (Hsieh *et al.* 1994). well-defined Melnikov function (Hsieh *et al.* 1994).<br>(*b*) *The Melnikov function* 

Melnikov's method is a way to determine what happens to the separatrices associated with the system's saddle points when the damping and forcing effects are added Melnikov's method is a way to determine what happens to the separatrices associated with the system's saddle points when the damping and forcing effects are added to the unperturbed system. Roughly speaking, it is a measur ated with the system's saddle points when the damping and forcing effects are added<br>to the unperturbed system. Roughly speaking, it is a measure of the separation of the<br>stable and unstable manifolds of the saddle point. T stable and unstable manifolds of the saddle point. These are coincident in the unper-<br>turbed system, but this situation does not persist when excitation and damping are added. In particular, the separation can be expressed as a function of the excitation turbed system, but this situation does not persist when excitation and damping are added. In particular, the separation can be expressed as a function of the excitation phase, specified by the initial time  $t_0$ , as  $d(t_0$ added. In particular, the separation can be expressed as a function of the excitation<br>phase, specified by the initial time  $t_0$ , as  $d(t_0) = \epsilon M(t_0) + O(\epsilon^2)$ , where  $M(t_0)$  is the<br>Melnikov function (Wiggins 1990). If the st phase, specified by the initial time  $t_0$ , as  $d(t_0) = \epsilon M(t_0) + O(\epsilon^2)$ , where  $M(t_0)$  is the Melnikov function (Wiggins 1990). If the stable manifold, in the face of the perturbations, remains 'outside' of the unstable man *Phil. Trans. R. Soc. Lond.* A (2000)

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nate of the separatrix  $y_0^+(t)$  for no heel,  $y_\delta(t)$ <br>  $\delta = 0.070$  for  $4^\circ$  heel,  $\delta = 0.140$  for  $8^\circ$  heel.

 $\delta = 0.070$  for 4° heel,  $\delta = 0.140$  for 8° heel.<br>in the sense that responses started inside the safe basin will remain stable, even if in the sense that responses started inside the safe basin will remain stable, even if<br>they are close to the basin boundary. In this case, the Melnikov function, by the sign<br>convention generally adopted will be negative for in the sense that responses started inside the safe basin will remain stable, even if<br>they are close to the basin boundary. In this case, the Melnikov function, by the sign<br>convention generally adopted, will be negative fo they are close to the basin boundary. In this case, the Melnikov function, by the sign<br>convention generally adopted, will be negative for all time. On the other hand, if the<br>stable manifold is 'inside' of the unstable mani convention generally adopted, will be negative for all time. On the other hand, if the stable manifold is 'inside' of the unstable manifold for all time, all solutions starting near the basin boundary will lead to capsize. Here, the Melnikov function is positive for all time. The more interesting case is when these manifolds cross one another as a function of time, continually switching relative near the basin boundary will lead to capsize. Here, the Melnikov function is positive<br>for all time. The more interesting case is when these manifolds cross one another as<br>a function of time, continually switching relative for all time. The more interesting case is when these manifolds cross one another as<br>a function of time, continually switching relative orientations. (This is the situation<br>when chaos appears (see Wiggins 1990).) In this c when chaos appears (see Wiggins 1990).) In this case, some solutions started near the boundary will escape, and this opens the door for the possibility of capsize even when chaos appears (see Wiggins 1990).) In this case, some solutions started near<br>the boundary will escape, and this opens the door for the possibility of capsize even<br>if the vessel starts in the safe basin. In fact, chaot the boundary will escape, and this opens the door for the possibility of capsize even<br>if the vessel starts in the safe basin. In fact, chaotic transport theory shows that<br>the area under the positive part of the Melnikov fu if the vessel starts in the safe basin. In fact, chaotic transport theory shows that<br>the area under the positive part of the Melnikov function is related to the rate at<br>which solutions, as measured by volumes of the phase the area under the positive part of the Melnikov function is related to the rate at<br>which solutions, as measured by volumes of the phase space, escape the safe basin.<br>This is quantified by a measure of the time-averaged fl which solutions, as measured by volumes of the phase space, escape the safe basin.<br>This is quantified by a measure of the time-averaged flux of phase space out of the<br>safe basin—a quantity that is related directly to the M below. safe basin—a quantity that is related directly to the Melnikov function, as described<br>below.<br>Knowledge of this flux rate allows one to obtain a quantitative measure for the

below.<br>Knowledge of this flux rate allows one to obtain a quantitative measure for the<br>likelihood of capsize for a given vessel and sea state. Descriptions of this technique<br>in the context of ship capsize can be found in t Knowledge of this flux rate allows one to obtain a quantitative measure for the likelihood of capsize for a given vessel and sea state. Descriptions of this technique in the context of ship capsize can be found in the lite likelihood of capsize for a given vessel and sea state. Descriptions of this technique<br>in the context of ship capsize can be found in the literature. The Melnikov theory for<br>the usual roll differential equation has been c in the context of ship capsize can be found in the literature. The Melnikov theory for<br>the usual roll differential equation has been considered in detail for both harmonic<br>and random excitation (Falzarano *et al.* 1992, Hs The usual roll differential equation has been considered in detail for both harmonic<br>and random excitation (Falzarano *et al.* 1992, Hsieh *et al.* 1994; Jiang *et al.* 1996).<br>However, the integro-differential equation re and random excitation (Falzarano *et al.* 1992, Hsieh *et al.* 1994; Jiang *et al.* 1996).<br>However, the integro-differential equation requires special treatment. This is due to<br>the fact that the integral term in the pertu However, the integro-differential equation requires special treatment. This is due to<br>the fact that the integral term in the perturbation does not depend simply on the<br>instantaneous values of the system states and time, bu the fact that the integral term in the perturbation does not depend simply on the instantaneous values of the system states and time, but on the history of the roll velocity. This complicates the analysis and the interpret instantaneous values of the system states and time, but on the history of the roll velocity. This complicates the analysis and the interpretation of the perturbation in a non-trivial way. However, using a standard approach non-trivial way. However, using a standard approach that approximates the memory<br>*Phil. Trans. R. Soc. Lond.* A (2000)

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effects by the output of a finite-dimensional linear dynamical system, one can modify effects by the output of a finite-dimensional linear dynamical system, one can modify<br>the Melnikov theory to handle these effects. The technical details of this analysis are<br>presented in Appendix A. The net outcome of the effects by the output of a finite-dimensional linear dynamical system, one can modify<br>the Melnikov theory to handle these effects. The technical details of this analysis are<br>presented in Appendix A. The net outcome of the the Melnikov theory to handle these effects. The technical details of this analysis are<br>presented in Appendix A. The net outcome of the results derived there is that one<br>can procedurally treat the integral perturbation ter presented in Appendix A. The net outcome of the results derived there is that one<br>can procedurally treat the integral perturbation term in the usual manner that has<br>been developed for perturbations of planar systems.<br>The M can procedurally treat the integral perturbation term in the usual manner that has been developed for perturbations of planar systems.

see Hsieh *et al.* 1994; Jiang *et al.* 1996)

$$
\begin{aligned} \text{Hsech } et \text{ al. } 1994; \text{ Jiang } et \text{ al. } 1996) \\ M_{\delta}(t_0) &= \int_{-\infty}^{\infty} y_{\delta}(t) \left[ -\int_{-\infty}^{\infty} \delta_{\rm m}(t-u) y_{\delta}(u) \, \mathrm{d}u - \delta_2 y_{\delta}(t) |y_{\delta}(t)| + f(t+t_0) \right] \, \mathrm{d}t \\ &= \tilde{M}_{\delta}(t_0) - \bar{M}_{\delta}, \end{aligned} \tag{3.7}
$$

where

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$$
\bar{M}_{\delta} = \int_{-\infty}^{\infty} y_{\delta}(t)\gamma_{\delta}(t) dt + \delta_2 \int_{-\infty}^{\infty} y_{\delta}(t)^2 |y_{\delta}(t)| dt, \qquad (3.8)
$$

$$
\tilde{M}_{\delta} = \int_{-\infty}^{+\infty} g_{\delta}(t) \, \rho_{\delta}(t) \, dt + \sigma_2 \int_{-\infty}^{+\infty} g_{\delta}(t) \, |g_{\delta}(t)| \, dt, \tag{3.9}
$$
\n
$$
\tilde{M}_{\delta}(t_0) = \int_{-\infty}^{\infty} g_{\delta}(t) f(t + t_0) \, dt \tag{3.9}
$$
\nare the constant and oscillatory parts of the Melnikov function, respectively, and

illatory parts of the Melnikov function, respectively, and  
\n
$$
\gamma_{\delta}(t, y) = \int_{-\infty}^{\infty} \delta_{\rm m}(t - u) y_{\delta}(u) \, \mathrm{d}u.
$$
\n(3.10)

For the unbiased case ( $\delta = 0$ ), the Melnikov function  $M_0(t_0)$  is defined along one For the unbiased case  $(\delta = 0)$ , the Melnikov function  $M_0(t)$  of the heteroclinic orbits, in which case (Hsieh *et al.* 1994),

hich case (Hsieh et al. 19)

\n
$$
M_0(t_0) = \tilde{M}_0(t_0) - \bar{M}_0,
$$

where

$$
\bar{M}_0 = \int_{-\infty}^{\infty} y_0^+(t)\gamma_0(t, y) dt + \delta_2 \int_{-\infty}^{\infty} (y_0^+(t))^2 |y_0(t)| dt,
$$
\n(3.11)

$$
\tilde{M}_0(t_0) = \int_{-\infty}^{\infty} y_0^+(t) f_0(t, y) dt + \frac{\omega_2}{2} \int_{-\infty}^{\infty} (y_0^-(t)) \left[ y_0(t) \right] dt,
$$
\n(3.11)\n
$$
\tilde{M}_0(t_0) = \int_{-\infty}^{\infty} y_0^+(t) f(t + t_0) dt.
$$
\n(3.12)

 $M_0(t_0) = \int_{-\infty} y_0'(t) f(t+t_0) dt.$  (3.12)<br>These results can be compared with those obtained from the standard differential<br>uation model. Of course, the unperturbed versions of (2.9) and (2.10) are iden-These results can be compared with those obtained from the standard differential<br>equation model. Of course, the unperturbed versions of  $(2.9)$  and  $(2.10)$  are iden-<br>tical. And even though the perturbed phase space is ve These results can be compared with those obtained from the standard differential<br>equation model. Of course, the unperturbed versions of  $(2.9)$  and  $(2.10)$  are iden-<br>tical. And, even though the perturbed phase space is v equation model. Of course, the unperturbed versions of  $(2.9)$  and  $(2.10)$  are identical. And, even though the perturbed phase space is very different for these two cases, the corresponding Melnikov functions have simila tical. And, even though the perturbed phase space is very different for these two cases, the corresponding Melnikov functions have similar form. In fact, the formu-<br>lae of the oscillatory parts are the same (i.e. equation cases, the corresponding Melnikov functions have similar form. In fact, the formu-<br>lae of the oscillatory parts are the same (i.e. equations (3.9) and (3.12)), but the<br>non-dimensional force  $f(t)$  is different from the dim lae of the oscillatory parts are the same (i.e. equations (3.9) and (3.12)), but the non-dimensional force  $f(t)$  is different from the dimensional excitation  $F(\tau)$ , simply because the total inertia used to scale  $\tau$  is C non-dimensional force  $f(t)$  is different from the dimensional excitation  $F(\tau)$ , simply<br>because the total inertia used to scale  $\tau$  is different. Here, the constant part of the<br>Melnikov function for (2.9) does not invo

$$
\bar{M}_{\delta} = \delta_1 \int_{-\infty}^{\infty} y_{\delta}(t)^2 dt + \delta_2 \int_{-\infty}^{\infty} y_{\delta}(t)^2 |y_{\delta}(t)| dt.
$$
 (3.13)

 $M_{\delta} = \delta_1 \int_{-\infty} y_{\delta}(t)^2 dt + \delta_2 \int_{-\infty} y_{\delta}(t)^2 |y_{\delta}(t)| dt.$  (3.13)<br>In order to calculate  $\overline{M}_{\delta}$  in (3.8),  $\gamma_{\delta}(t, y)$  in (3.10) must first be determined. It<br>recognized from (3.10) that  $\gamma_{\delta}(t, y)$  is the convolution is in order to calculate  $\overline{M}_{\delta}$  in (3.8),  $\gamma_{\delta}(t, y)$  in (3.10) must first be determined. It<br>is recognized from (3.10) that  $\gamma_{\delta}(t, y)$  is the convolution of  $K(t)$  and  $y_{\delta}(t)$ . It can<br>be numerically evaluated by In order to calculate  $\overline{M}_{\delta}$  in (3.8),  $\gamma_{\delta}(t, y)$  in (3.10) must first be determined. It is recognized from (3.10) that  $\gamma_{\delta}(t, y)$  is the convolution of  $K(t)$  and  $y_{\delta}(t)$ . It can be numerically evaluated by a c be numerically evaluated by a combination of fast Fourier transform and the inverse<br>*Phil. Trans. R. Soc. Lond.* A (2000)



Figure 6. The intermediate function  $\gamma_{\delta}(t, y)$ .  $\delta = 0.070$  for 4<sup>°</sup> heel,  $\delta = 0.140$  for 8<sup>°</sup> heel.

figure 6. The intermediate function  $\gamma_{\delta}(t, y)$ ,  $\delta = 0.070$  for 4 heel,  $\delta = 0.140$  for 8 heel.<br>fast Fourier transform (Bracewell 1978). The results for different values of the heel<br>angle are presented in figure 6. Unli fast Fourier transform (Bracewell 1978). The results for different values of the heel<br>angle are presented in figure 6. Unlike the constant coefficient representation, where<br>the linear damping  $\overline{M}_s$  changes proportiona fast Fourier transform (Bracewell 1978). The results for different values of the heel<br>angle are presented in figure 6. Unlike the constant coefficient representation, where<br>the linear damping  $\overline{M}_{\delta}$  changes proportio angle are presented in figure 6. Unlike the constant coefficient representation, where<br>the linear damping  $\bar{M}_{\delta}$  changes proportionally with the linear damping coefficient<br>(equation (3.13)), the memory-function repres the linear damping  $M_{\delta}$  changes proportionally with the linear damping coefficient (equation (3.13)), the memory-function representation of the linear hydrodynamic radiation influences  $\overline{M}_{\delta}$  in a complex manner t  $\delta$   $\overline{a}$ (equation  $(3.13)$ <br>radiation influe:<br>further in  $\S 4$ .

## (*c*) *Some features of the Melnikov function*

The constant part of the Melnikov function depends directly on the damping coef- The constant part of the Melnikov function depends directly on the damping coef-<br>ficients, while the oscillating part is linearly related to the excitation. The complete<br>Melnikov function describes the relative orientatio The constant part of the Melnikov function depends directly on the damping coef-<br>ficients, while the oscillating part is linearly related to the excitation. The complete<br>Melnikov function describes the relative orientatio Melnikov function describes the relative orientation of the stable and unstable manifolds of the saddle point(s) at the angle(s) of vanishing stability as a function of Melnikov function describes the relative orientation of the stable and unstable manifolds of the saddle point(s) at the angle(s) of vanishing stability as a function of time (via the forcing phase  $t_0$ ). For an unforced if olds of the saddle point(s) at the angle(s) of vanishing stability as a function of<br>time (via the forcing phase  $t_0$ ). For an unforced system with non-zero damping, the<br>Melnikov function is a negative constant. This i time (via the forcing phase  $t_0$ ). For an unforced system with non-zero damping, the Melnikov function is a negative constant. This implies that the unstable manifolds of the saddle points at the angles of vanishing stab Melnikov function is a negative constant. This implies that the unstable manifolds of<br>the saddle points at the angles of vanishing stability lie inside the corresponding sta-<br>ble manifolds (Guckenheimer & Holmes 1983; Wig the saddle points at the angles of vanishing stability lie inside the corresponding stable manifolds (Guckenheimer & Holmes 1983; Wiggins 1990). In this case, all initial conditions located inside the safe basin of the as ble manifolds (Guckenheimer & Holmes 1983; Wiggins 1990). In this case, all initial conditions located inside the safe basin of the associated undamped system will lead to motions that approach the stable upright position Conditions located inside the safe basin of the associated undamped system will lead<br>to motions that approach the stable upright position of the ship as  $t \to \infty$ . As the<br>nexcitation amplitude is increased from zero, the M to motions that approach the stable upright position of the ship as  $t \to \infty$ . As the excitation amplitude is increased from zero, the Melnikov function starts to oscillate about its mean, with an amplitude proportional to excitation amplitude is increased from zero, the Melnikov function starts to oscillate<br>about its mean, with an amplitude proportional to the excitation level. When the<br>forcing is sufficiently large that the Melnikov functi about its mean, with an amplitude proportional to the excitation level. When the forcing is sufficiently large that the Melnikov function crosses zero and is therefore positive for some duration of the excitation, the unst forcing is sufficiently large that the Melnikov function crosses zero and is therefore<br>positive for some duration of the excitation, the unstable manifold lies outside of<br>the stable manifold, and solutions between them wil positive for some duration of the excitation, the unstable manifold lies outside of the stable manifold, and solutions between them will be transported out of the safe region. These regions between the manifolds are known region. These regions between the manifolds are known as 'lobes' (Wiggins 1992).<br>*Phil. Trans. R. Soc. Lond.* A (2000)

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When lobes exist, the situation exists in which some solutions with initial conditions near the separatrix will escape to capsize.

For Gaussian excitation, the Melnikov function will have some zero crossings for near the separatrix will escape to capsize.<br>For Gaussian excitation, the Melnikov function will have some zero crossings for<br>any non-zero level of wave forcing, implying that there is a non-zero probability for<br>capsizing a For Gaussian excitation, the Melnikov function will have some zero crossings for<br>any non-zero level of wave forcing, implying that there is a non-zero probability for<br>capsizing as soon as the wave forcing is introduced. On capsizing as soon as the wave forcing is introduced. One consequence of this situation<br>is that the dynamics of the system started near the boundary of the safe basin are essentially unpredictable, and capsize may occur (Thomson 1989b; Falzarano *et* is that the dynamics of the system started near the boundary of the safe basin<br>are essentially unpredictable, and capsize may occur (Thomson 1989*b*; Falzarano *et*<br>*al.* 1992). In fact, for the case of harmonic excitatio are essentially unpredictable, and capsize may occur (Thomson 1989*b*; Falzarano *et al.* 1992). In fact, for the case of harmonic excitation, the safe basin boundary has a fractal nature (Moon & Li 1985). In both the har a fractal nature (Moon  $&$  Li 1985). In both the harmonic and random excitation cases, the likelihood of an initial condition escaping the safe region depends on a a fractal nature (Moon & Li 1985). In both the harmonic and random excitation cases, the likelihood of an initial condition escaping the safe region depends on a measure of the positive part of the Melnikov function. This cases, the likelihood of an initial condition escaping the safe region depends on a measure of the positive part of the Melnikov function. This can be quantified by knowing the mean frequency of the zero crossings of  $M(t_$ measure of the positive part of the Melnikov function. This can be quantified by knowing the mean frequency of the zero crossings of  $M(t_0)$ , the percentage of time that it is positive, and its positive amplitude. These, knowing the mean frequency of the zero crossings of  $M(t_0)$ , the percentage of time<br>that it is positive, and its positive amplitude. These, in turn, depend on the level<br>of dissipation, through the mean value of  $M(t_0)$ , a that it is positive, and its positive amplitude. These, in turn, depend on the level<br>of dissipation, through the mean value of  $M(t_0)$ , and on the overall amplitude and<br>frequency content of the excitation, which dictate t of dissipation, through the mean value of  $M(t_0)$ , and on the overall amplitude and frequency content of the excitation, which dictate the oscillatory part of  $M(t_0)$ . Our goal here is to obtain a measure of these quantit goal here is to obtain a measure of these quantities and relate them to the likelihood

### (*d*) The useful statistics of  $M(t_0)$

(d) The useful statistics of  $M(t_0)$ <br>For the case of random seas, the oscillatory part of the Melnikov function  $\tilde{M}_{\delta}(t_0)$ <br>a stochastic process. The results for this term are the same for both the differ-For the case of random seas, the oscillatory part of the Melnikov function  $\tilde{M}_{\delta}(t_0)$  is a stochastic process. The results for this term are the same for both the differential and integro-differential system models, For the case of random seas, the oscillatory part of the Melnikov function  $M_{\delta}(t_0)$  is a stochastic process. The results for this term are the same for both the differential and integro-differential system models, sinc is a stochastic process. The results for this te<br>ential and integro-differential system models,<br>value of  $M(t_0)$ . The correlation between  $\tilde{M}_{\delta}(t_0)$ <br>volution integral indicated in (3.9) and (3.12) is term are the same for both the differ-<br>ls, since damping affects only the mean<br> $\delta(t_0)$  and  $f(t)$  is linear, through the con-<br>12). Using the evenness and oddness of ential and integro-differential system models, since damping affects only the mean<br>value of  $M(t_0)$ . The correlation between  $\tilde{M}_{\delta}(t_0)$  and  $f(t)$  is linear, through the con-<br>volution integral indicated in (3.9) and ( value of  $M(t_0)$ . The correlation between  $M_\delta(t_0)$  and  $f(t)$  is linear, through the convolution integral indicated in (3.9) and (3.12). Using the evenness and oddness of the roll velocity taken along the basin boundaries volution integral indicate<br>the roll velocity taken alc<br>transformed as follows:

transformed as follows:  
\n
$$
\tilde{M}_{\delta}(t_0) = \int_{-\infty}^{\infty} -y_{\delta}(t_0 - t)f(t) dt
$$
\n(3.14)  
\nfor the homoclinic case, and similarly for the heteroclinic case,

for the homoclinic case, and similarly for the heteroclinic case,  
\n
$$
\tilde{M}_0(t_0) = \int_{\infty}^{\infty} y_0^+(t_0 - t) f(t) dt.
$$
\n(3.15)  
\nIf  $f(t)$  is a zero mean random process, then the expected value of  $\tilde{M}_0(t_0)$  is also zero,

If 
$$
f(t)
$$
 is a zero mean random process, then the expected value of  $\tilde{M}_0(t_0)$  is also zero,

s, then the expected value of 
$$
\tilde{M}_0(t_0)
$$
 is also zero,  

$$
E[\tilde{M}_\delta] = 0.
$$
(3.16)

 $E[\tilde{M}_{\delta}] = 0.$  (3.16)<br>Under the assumption that  $f(t)$  is stationary, the spectrum of  $\tilde{M}_{\delta}(t_0)$  is given by

Under the assumption that 
$$
f(t)
$$
 is stationary, the spectrum of  $\tilde{M}_{\delta}(t_0)$  is given by  
\n
$$
S_{\tilde{M}_{\delta}}^{+}(\Omega) = (2\pi)^2 |Y_{\delta}(\Omega)|^2 S_{\rm f}^{+}(\Omega) \qquad (3.17)
$$
\n
$$
= (2\pi)^2 |Y_{\delta}(\Omega)|^2 |F_{\rm roll}(\Omega)|^2 S_{\zeta}^{+}(\Omega), \qquad (3.18)
$$
\nwhere  $Y_{\delta}(\Omega)$  is the Fourier transform of  $y_{\delta}(t)$ ,

$$
= (2\pi)^{2} |Y_{\delta}(\Omega)|^{2} |F_{\text{roll}}(\Omega)|^{2} S_{\zeta}^{+}(\Omega), \qquad (3.18)
$$

$$
Y_{\delta}(\Omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} y_{\delta}(t) e^{-i\Omega t} dt,
$$
\n(3.19)

and  $S_{\rm f}^+(\Omega)$  $Y_{\delta}(M)$ <br>f ( $\Omega$ ),  $F_{\text{roll}}(\Omega)$  and  $S_{\zeta}^{+}(\Omega)$ <br>taxe height and incident w  $+\omega$  $\delta(S) = \frac{1}{2\pi} \int_{-\infty} y_{\delta}(t) e^{-i\omega t} dt,$  (3.19)<br>  $\frac{1}{\zeta}(\Omega)$  are the scaled forcing spectrum, roll moment per<br>
int wave spectrum respectively and  $S_{\rm f}^+(\Omega)$ ,  $F_{\rm roll}(\Omega)$  and  $S_{\zeta}^+(\Omega)$  are the scaled forcing spectrum wave height and incident wave spectrum, respectively. *Phil. Trans. R. Soc. Lond.* A (2000)

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The Gaussian and ergodic natures of a random process are preserved under linear transformations. Thus, if the random wave elevation  $\zeta(t)$  is a stationary ergodic Gaussian random process, then the roll excitation  $f(t)$  a **MATHEMATICAL,<br>PHYSICAL<br>& ENGINEERING<br>SCIENCES** The Gaussian and ergodic natures of a random process are preserved under linear transformations. Thus, if the random wave elevation  $\zeta(t)$  is a stationary ergodic Gaussian random process, then the roll excitation  $f(t)$  a The Gaussian and ergodic natures of a random process are preserved under linear transformations. Thus, if the random wave elevation  $\zeta(t)$  is a stationary ergodic<br>Gaussian random process, then the roll excitation  $f(t)$  and the oscillatory part of Mel-<br>nikov function  $\tilde{M}_{\delta}(t_0)$  will share th Gaussian random process, then the roll excitation  $f(t)$  and the oscillatory part of Melnikov function  $\tilde{M}_{\delta}(t_0)$  will share those properties, since they are all linearly related to one another. For this stationary ra nikov function  $M_{\delta}(t_0)$  will share those properties, since they are all linearly related autocorrelation function and the power spectrum are related as follows:

$$
R_{\tilde{M}_{\delta}}(s,t) = R_{\tilde{M}_{\delta}}(s-t)
$$
  
\n
$$
= R_{\tilde{M}_{\delta}}(\tau)
$$
  
\n
$$
= \int_0^\infty S_{\tilde{M}_{\delta}}^+(\Omega) e^{-i\Omega \tau} d\Omega,
$$
  
\n
$$
E[\tilde{M}_{\delta}^2(t_0)] = R_{\tilde{M}_{\delta}}(0)
$$
\n(3.20)

$$
E[\tilde{M}_{\delta}^{2}(t_{0})] = R_{\tilde{M}_{\delta}}(0)
$$
  
= 
$$
\int_{0}^{\infty} S_{\tilde{M}_{\delta}}^{+}(\Omega) d\Omega,
$$
 (3.21)

 $=\int_0^{\infty} S_{\tilde{M}_\delta}^{\perp}(I) dI,$  (3.21)<br>where  $\tau = s - t$  and  $R_{\tilde{M}_\delta}(s, t)$  is a function of  $\tau$  only because  $\tilde{M}_\delta(t_0)$  is stationary.<br>By using (3.18) (3.16) (3.17) and (3.21) one can obtain the following relations where  $\tau = s - t$  and  $R_{\tilde{M}_{\delta}}(s, t)$  is a function of  $\tau$  only because  $\tilde{M}_{\delta}(t_0)$  is stationary.<br>By using (3.18), (3.16), (3.17) and (3.21), one can obtain the following relationship<br>between the variance of  $\tilde{M}_{$ where  $\tau = s - t$  and  $R_{\tilde{M}_\delta}(s, t)$  is a function of  $\tau$  only because  $M_\delta(t_0)$  is s<br>By using (3.18), (3.16), (3.17) and (3.21), one can obtain the following re<br>between the variance of  $\tilde{M}_\delta(t_0)$  and the spectrum of

$$
\sigma_{\tilde{M}_{\delta}}^2 = E[\tilde{M}_{\delta}^2(t_0)]
$$
  
= 
$$
\int_0^\infty (2\pi)^2 |Y_{\delta}(\Omega)|^2 |F_{\text{roll}}(\Omega)|^2 S_{\zeta}^+(\Omega) d\Omega.
$$
 (3.22)

Because  $\tilde{M}_{\delta}(t_0)$  $\begin{align*}\n\overline{\mathcal{L}}_{\delta}(t_0) & \text{is a stationary Gaussian process, it is uniquely determined by its}\n\end{align*}$ <br>
Individual variance  $\sigma^2$ . The zero-mean random variable  $x = \tilde{M}_s(t_0)$  has the folmean  $\bar{M}_\delta$  ar  $\tilde{M}_{\delta}(t_0)$  is a station<br>  $\delta$  and variance  $\sigma^2_{\tilde{M}_{\delta}}$ .<br>
robability density fi tionary Gaussian process, it is uniquely determined by its  $\tilde{M}_{\delta}$ . The zero-mean random variable  $x = \tilde{M}_{\delta}(t_0)$  has the fol-<br>ty function at any value of  $t_0$ . Because  $M_{\delta}(t_0)$  is a stationary Gaussian process, it is<br>mean  $\bar{M}_{\delta}$  and variance  $\sigma^2_{\tilde{M}_{\delta}}$ . The zero-mean random vari<br>lowing probability density function at any value of  $t_0$ : lowing probability density function at any value of  $t_0$ :

$$
p_{\tilde{M}_{\delta}}(x) = \frac{1}{\sqrt{2\pi}\sigma_{\tilde{M}_{\delta}}} \exp\left(-\frac{x^2}{2\sigma_{\tilde{M}_{\delta}}^2}\right).
$$
 (3.23)

 $P_{M_{\delta}}(x) = \frac{1}{\sqrt{2\pi}\sigma_{\tilde{M}_{\delta}}} \exp\left(-\frac{1}{2\sigma_{\tilde{M}_{\delta}}^2}\right)$ . (3.23)<br>This Gaussian structure will yield an analytical measure for the positive part of the<br>Melnikov function This Gaussian struct<br>Melnikov function. (*e*) *Phase-space transport and the Melnikov function*

The amount of phase space transported out of the safe region is related to the The amount of phase space transported out of the safe region is related to the<br>areas of the lobes formed when the stable manifold is inside of the unstable mani-<br>fold i.e. where  $M(t_0) > 0$  (Recall that these are the situa The amount of phase space transported out of the safe region is related to the areas of the lobes formed when the stable manifold is inside of the unstable manifold, i.e. where  $M(t_0) > 0$ . (Recall that these are the situa areas of the lobes formed when the stable manifold is inside of the unstable manifold, i.e. where  $M(t_0) > 0$ . (Recall that these are the situations for which solutions can escape the safe region.) This area can be approxi fold, i.e. where  $M(t_0) > 0$ . (Recall that these are the situations for which solutions<br>can escape the safe region.) This area can be approximated in asymptotic form by<br>integrating the Melnikov function over those times fo can escape the safe region.) This area can be approximated in asymptotic form by<br>integrating the Melnikov function over those times for which it is positive (Wiggins<br>1992). For a random process, the following sum of integr integrating the Melnikov function over those times for which it is positive (Wiggins 1992). For a random process, the following sum of integrals of the Melnikov function provides the desired measure of the area of phase s 1992). For a random process, the following sum of integral provides the desired measure of the area of phase space region over a given time interval (Frey  $\&$  Simiu 1993): region over a given time interval (Frey  $\&$  Simiu 1993):

$$
\mu = \epsilon \sum_{i} \int_{t_{i1}}^{t_{i2}} M_{\delta}(t_0) dt_0 + O(\epsilon^2), \tag{3.24}
$$

where  $[t_{i1}, t_{i2}]$  is the *i*th interval over which  $M_{\delta}(t_0) > 0$ .

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THE ROYAL

**PHILOSOPHICAL<br>TRANSACTIONS** 

**MATHEMATICAL,<br>PHYSICAL**<br>& ENGINEERING

LOYAL<br>ETY

In order to have a more useful measure of the likelihood of the escape of solutions out of the safe basin under random excitation, we take the long-time average of this quantity, resulting in a rate of phase-space flux, a **HYSICAL**<br>· ENGINEERING<br>CIENCES In order to have a more useful measure of the likelihood of the escape of solutiout of the safe basin under random excitation, we take the long-time average of quantity, resulting in a rate of phase-space flux, as follows In order to have a more useful measure of the likelihood of the escape of solutions

g in a rate of phase-space flux, as follows (Hsien *et al.* 1994):  
\n
$$
\Phi_{\delta} = \lim_{T \to \infty} \frac{\epsilon}{2T} \int_{-T}^{T} M_{\delta}^{+}(t_0) dt_0 + O(\epsilon^2)
$$
\n
$$
= \lim_{T \to \infty} \frac{\epsilon}{2T} \int_{-T}^{T} (\tilde{M}_{\delta}(t_0) - \bar{M}_{\delta})^{+} dt_0 + O(\epsilon^2), \qquad (3.25)
$$

where  $M_{\delta}^+(t_0)$  $=\lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T} (M_\delta(t_0)-M_\delta)^T d\tau_0+O(\epsilon^2),$  (3.25)<br>  $\int_{\delta}^{+}(t_0)$  denotes the positive part of the Melnikov function. Since  $M_\delta(t_0)$  is equal to its interactionary Gaussian process, the time average of  $M_\delta(t_0$ where  $M_{\delta}^{+}(t_0)$  denotes the positive part of the Melnikov function. Since  $M_{\delta}(t_0)$  is an ergodic stationary Gaussian process, the time average of  $M_{\delta}(t_0)$  is equal to its ensemble average. where  $M_{\delta}^{+}(t_0)$  denc<br>an ergodic stational<br>ensemble average,

ensemble average,  
\n
$$
\lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} M_{\delta}^{+}(t_0) dt_0 = E[M_{\delta}^{+}(t_0)]
$$
\n
$$
= E[(\tilde{M}_{\delta}(t_0) - \bar{M}_{\delta})^{+}].
$$
\nTherefore, the rate of phase-space flux becomes, to leading order,

phase-space flux becomes, to leading order,  
\n
$$
\Phi_{\delta} = \epsilon E[(\tilde{M}_{\delta}(t_0) - \bar{M}_{\delta})^+] + O(\epsilon^2)
$$
\n(3.26)

$$
= \epsilon E[(\tilde{M}_{\delta}(t_0) - \bar{M}_{\delta})^+] + O(\epsilon^2)
$$
\n
$$
= \epsilon \int_{\bar{M}_{\delta}}^{\infty} (x - \bar{M}_{\delta}) p_{\tilde{M}_{\delta}}(x) dx + O(\epsilon^2),
$$
\n(3.27)

 $= \epsilon \int_{\bar{M}_{\delta}} (x - M_{\delta}) p_{\tilde{M}_{\delta}}(x) dx + O(\epsilon^2),$  (3.27)<br>where x is a random variable representing the Gaussian random process  $\tilde{M}_{\delta}(t_0)$ , with<br>zero mean, variance  $\sigma^2$ , and the PDF given in (3.23) where x is a random varia<br>zero mean, variance  $\sigma_{\tilde{M}_{\delta}}^2$ <br>The standard Gaussian  $\frac{2}{M_s}$ <br>ariable representing the Gaussian rate<br> $\frac{2}{M_s}$  and the PDF given in (3.23).<br>sian probability density function *n*( here x is a random variable representing the Gaussian random process  $M_{\delta}(t_0)$ , with<br>ro mean, variance  $\sigma_{\tilde{M}_{\delta}}^2$  and the PDF given in (3.23).<br>The standard Gaussian probability density function  $p(z)$  and the assoc

zero mean, variance  $\sigma_{\tilde{M}_{\delta}}^2$  and the PDF given in (3.23).<br>The standard Gaussian probability density function  $p(z)$  ability distribution function  $P(z)$  are related as follows: ability distribution function  $P(z)$  are related as follows:

$$
p(z) = \frac{1}{\sqrt{2\pi}} \exp(-\frac{1}{2}z^2),
$$
\n(3.28)

$$
p(z) = \frac{1}{\sqrt{2\pi}} \exp(-\frac{1}{2}z^2),
$$
\n
$$
P(z) = \int_{-\infty}^{z} p(x) dx.
$$
\n(3.29)

Using these relationships, equation (3.27) can be written as

$$
\Phi_{\delta} = \epsilon \int_{\bar{M}_{\delta}}^{\infty} (x - \bar{M}_{\delta}) \frac{1}{\sqrt{2\pi}\sigma_{\tilde{M}_{\delta}}} \exp\left(-\frac{x^{2}}{2\sigma_{\tilde{M}_{\delta}}^{2}}\right) dx + O(\epsilon^{2})
$$
\n
$$
= \epsilon \sigma_{\tilde{M}_{\delta}} \int_{\bar{M}_{\delta}/\sigma_{\tilde{M}_{\delta}}}^{\infty} \frac{1}{\sqrt{2\pi}} \left(z - \frac{\bar{M}_{\delta}}{\sigma_{\tilde{M}_{\delta}}}\right) \exp(-\frac{1}{2}z^{2}) dz + O(\epsilon^{2})
$$
\n
$$
= \epsilon \left[\sigma_{\tilde{M}_{\delta}} p\left(\frac{\bar{M}_{\delta}}{\sigma_{\tilde{M}_{\delta}}}\right) + \bar{M}_{\delta} P\left(\frac{\bar{M}_{\delta}}{\sigma_{\tilde{M}_{\delta}}}\right) - \bar{M}_{\delta}\right] + O(\epsilon^{2}) \tag{3.30}
$$

for the biased case, and as

for the biased case, and as  
\n
$$
\Phi_{\text{upper}} = \epsilon \left[ \sigma_{\tilde{M}_0} p \left( \frac{\bar{M}_0}{\sigma_{\tilde{M}_0}} \right) + \bar{M}_0 P \left( \frac{\bar{M}_0}{\sigma_{\tilde{M}_0}} \right) - \bar{M}_0 \right] + O(\epsilon^2)
$$
\n(3.31)  
\nfor the unbiased case. Here,  $\Phi_{\text{upper}}$  is the rate of phase-space flux through the upper  
\nheteroclinic orbit

for the unbiased cas<br>heteroclinic orbit. *Phil. Trans. R. Soc. Lond.* A (2000)

**MATHEMATICAL,<br>PHYSICAL<br>& ENGINEERING** 

PHILOSOPHICAL THE ROYAL

**ATHEMATICAL** 

**MATHEMATICAL,<br>PHYSICAL<br>& ENGINEERING<br>SCIENCES** 

**PHILOSOPHICAL**<br>TRANSACTIONS

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**PHILOSOPHICAL**<br>TRANSACTIONS

76<br>If we denote  $\sigma_{\delta}^1$  as<br>te that  $\sigma_{\delta}$  is pro  $\frac{1}{3}$  as C. Jiang, A. W. Troesch and  $\check{S}$ . W. Shaw<br> $\frac{1}{\delta}$  as the RMS value of  $\tilde{M}_{\delta}(t_0)$  for unit significant wave height, and<br>proportional to the significant wave height  $H_{\alpha}$ ,  $\Phi_{\delta}$  can be rewritten If we denote  $\sigma_{\delta}^1$  as the RMS value of  $\tilde{M}_{\delta}(t_0)$  for unit significant wave height, and note that  $\sigma_{\tilde{M}_{\delta}}$  is proportional to the significant wave height  $H_s$ ,  $\Phi_{\delta}$  can be rewritten in a form normalize note that  $\sigma_{\tilde{M}_{\delta}}$  is proportional to the significant wave height  $H_s$ ,  $\Phi_{\delta}$  can be rewritten in a form normalized by the area of the unperturbed safe basin, as follows:

$$
\frac{\Phi_{\delta}}{A_{\delta}} = \frac{\epsilon}{A_{\delta}} \left[ H_{\rm s} \sigma_{\delta}^{1} p \left( \frac{\bar{M}_{\delta}}{H_{\rm s} \sigma_{\delta}^{1}} \right) + \bar{M}_{\delta} P \left( \frac{\bar{M}_{\delta}}{H_{\rm s} \sigma_{\delta}^{1}} \right) - \bar{M}_{\delta} \right] + O(\epsilon^{2}). \tag{3.32}
$$
  
The dependence of the flux rate  $\Phi_{\delta}/A_{\delta}$  in (2.10) on the significant wave height  
for several values of bias, at a characteristic wave period  $T_{\rm s} = 9.0$  s, is shown

 $H_{\delta}$   $H_{\delta}$   $H_{\delta}$   $H_{\delta}$   $\left\{\right.$   $\left\{\right.$   $H_{\delta} \circ \overline{\delta} \right\}$   $\left\{\right.$   $H_{\delta} \circ \overline{\delta} \right\}$   $\left\{\right.$  The dependence of the flux rate  $\Phi_{\delta}/A_{\delta}$  in (2.10) on the significant wave height  $H_{\delta}$  for several values The dependence of the flux rate  $\Phi_{\delta}/A_{\delta}$  in (2.10) on the significant wave height  $H_s$  for several values of bias, at a characteristic wave period  $T_s = 9.0$  s, is shown in figure 7. Note that there is a finite amount  $H_s$  for several values of bias, at a characteristic wave period  $T_s = 9.0$  s, is shown<br>in figure 7. Note that there is a finite amount of phase-space flux for any non-<br>zero excitation. This means that there is a finite pr in figure 7. Note that there is a finite amount of phase-space flux for any non-<br>zero excitation. This means that there is a finite probability of capsize for any non-<br>zero wave height, although for small wave heights, cap zero excitation. This means that there is a finite probability of capsize for any non-<br>zero wave height, although for small wave heights, capsize is highly unlikely over<br>any reasonable exposure time. The flux rate is very zero wave height, although for small wave heights, capsize is highly unlikely over<br>any reasonable exposure time. The flux rate is very small for small wave heights,<br>but begins to grow significantly beyond a critical wave any reasonable exposure time. The flux rate is very small for small wave heights,<br>but begins to grow significantly beyond a critical wave height, after which it grows<br>steadily as  $H_s$  increases, eventually approaching a l but begins to grow significantly beyond a c<br>steadily as  $H_s$  increases, eventually approac<br>is achieved as  $H_s \to \infty$ , and is given by

$$
\frac{\Phi_{\delta}}{A_{\delta}} \approx \frac{\epsilon}{A_{\delta}} \left( \frac{1}{\sqrt{2\pi}} H_{\rm s} \sigma_{\delta}^{1} - \frac{1}{2} \bar{M}_{\delta} \right). \tag{3.33}
$$

 $rac{\Psi_{\delta}}{A_{\delta}} \approx \frac{\epsilon}{A_{\delta}} \left( \frac{1}{\sqrt{2\pi}} H_{\rm s} \sigma_{\delta}^1 - \frac{1}{2} \bar{M}_{\delta} \right).$  (3.33)<br>The asymptote, shown in figure 7 for three values of the heel angle, intersects the  $H_{\rm s}$ -axis at a wave height given by  $A_{\delta}$   $A_{\delta}$  \The asymptote, shown in figure 7 for  $H_s$ -axis at a wave height given by

$$
H_{\rm s}^* = \frac{\sqrt{2\pi}\bar{M}_{\delta}}{2\sigma_{\delta}^1}.
$$
\n(3.34)

 $H_s^* = \frac{\sqrt{2h_0 h_0}}{2\sigma_{\delta}^1}$ . (3.34)<br>We define  $H_s^*$  as the critical wave height at which substantial phase-space flux begins<br>to occur, suggesting an increased risk of operating the ship. It provides a measure We define  $H_s^*$  as the critical wave height at which substantial phase-space flux begins<br>to occur, suggesting an increased risk of operating the ship. It provides a measure<br>of the combined effects of the large-amplitude to occur, suggesting an increased risk of operating the ship. It provides a measure of the combined effects of the large-amplitude roll characteristics of the vessel, the to occur, suggesting an increased risk of operating the ship. It provides a measure<br>of the combined effects of the large-amplitude roll characteristics of the vessel, the<br>amount of dissipation present and the nature of th of the combined effects of the large-amplitude roll characteristics of the vessel, the amount of dissipation present and the nature of the wave excitation in a relatively simple way. Figure 8 shows how  $H_s^*$  varies with amount of dissip<br>simple way. Figure<br>the heel angle.<br>As pointed out simple way. Figure 8 shows how  $H_s^*$  varies with the characteristic wave period and the heel angle.<br>As pointed out by Hsieh *et al.* (1994), the exact quantitative relation of the rate of

phase-space flux  $\Phi_{\delta}$  and the likelihood of capsize depends on the distribution of the As pointed out by Hsieh *et al.* (1994), the exact quantitative relation of the rate of phase-space flux  $\Phi_{\delta}$  and the likelihood of capsize depends on the distribution of the response in the phase space, the location phase-space flux  $\Phi_{\delta}$  and the likelihood of capsize depends on the distribution of the response in the phase space, the location of the phase space being transported, the replenishing of phase space into the safe basi response in the phase space, the location of the phase space being transported, the replenishing of phase space into the safe basin from the unsafe area and the exposure time. The details of such relationships are present replenishing of phase space into the safe basin from the unsafe area and the exposure time. The details of such relationships are presently unknown. However, simulation results presented by Hsieh *et al.* (1994) demonstrat time. The details of such relationships are presently unknown. However, simulation<br>results presented by Hsieh *et al.* (1994) demonstrated a high correlation between<br> $\Phi_{\delta}$  and the probability of capsize. We use  $H_{\delta}^{$ results presented by Hsieh *et al.* (1994) demonstrated a high correlation between  $\Phi_{\delta}$  and the probability of capsize. We use  $H_s^*$  as the critical significant wave height because, graphically, it represents the sta  $\Phi_{\delta}$  and the probability of capsize. We use  $H_{\rm s}^*$  as the critical significance, graphically, it represents the starting point where the radius flux increases virtually linearly with the significant wave height.<br>I cause, graphically, it represents the starting point where the rate of phase-space  $x$  increases virtually linearly with the significant wave height.<br>If one deems that  $H_s^*$  corresponds to a capsize probability too high

flux increases virtually linearly with the significant wave height.<br>If one deems that  $H_s^*$  corresponds to a capsize probability too high (or too low) to<br>be acceptable, one can calculate a different criterion. For exampl If one deems that  $H_s^*$  corresponds to a capsize probability too high (or too low) to<br>be acceptable, one can calculate a different criterion. For example, one can define a<br>certain rate of phase-space flux, and compute th be acceptable, one can calculate a different criterion. For example, one can define a certain rate of phase-space flux, and compute the associated significant wave height for different characteristic wave periods, using certain rate of phase-space flux, and compute the associated significant wave height<br>for different characteristic wave periods, using (3.32). Critical wave height curves<br>computed in this manner take similar shapes to the TH<sub>3</sub> for different characteristic wave periods, using  $(3.32)$ . Critical wave height curves<br>computed in this manner take similar shapes to the shape of the  $H_s^*$  curve. However,<br>they differ since the  $H_s^*$  curve does not mai computed in this manner take similar shapes to the shape of the  $H_s^*$  curve. However, they differ since the  $H_s^*$  curve does not maintain a constant rate of phase-space flux as the wave period varies. The results for th they differ since the  $H_s^*$  curve does not maintain a constant rate of phase-space flux as<br>the wave period varies. The results for the *Patti-B* are shown in figures 9, 10 and 11<br>for a 0, 4 and 8<sup>°</sup> bias angle, respectiv the wave period varies. The results for the *Patti-B* are shown in figures 9, 10 and 11 for a 0, 4 and 8° bias angle, respectively. Also note that the rates of phase-space flux at  $H_s^*$  change significantly with the bias period varies. The results for the *Patti-B* are shown in figures 9, 10 and 11<br>4 and 8° bias angle, respectively. Also note that the rates of phase-space<br> $\frac{k}{s}$  change significantly with the bias angle. In the unbiased for a 0, 4 and 8° bias angle, respectively. Also note that the rates of phase-space flux at  $H_s^*$  change significantly with the bias angle. In the unbiased case, the rate of phase-space flux at  $H_s^*$  is approximately 0. flux at  $H_s^*$  change significantly with the bias characteristic phase-space flux at  $H_s^*$  is approximately to for the 4 and 8<sup>°</sup> bias cases, respectively. *Phil. Trans. R. Soc. Lond.* A (2000)

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significant wave height (m)<br>Figure 7. The rate of phase-space flux at 9.0 s characteristic wave period considering memory<br>effects  $2\Phi_{\text{max}}/4s$  for the zero heel and  $\Phi_s/4s$  for the rest. The short broken lines are the Figure 7. The rate of phase-space flux at 9.0 s characteristic wave period considering memory effects,  $2\Phi_{\text{upper}}/A_0$  for the zero heel and  $\Phi_{\delta}/A_{\delta}$  for the rest. The short broken lines are the linear asymptotes asymptotes.



 $* \mathbf{A}$ 



wave period (s)<br>Figure 9. The significant wave height at  $0^{\circ}$  heel corresponding to equal level of phase-space flux<br>compared with the critical significant wave height  $H^*$  'BoPSE' refers to the rate of phase-space Figure 9. The significant wave height at  $0^{\circ}$  heel corresponding to equal level of phase-space flux compared with the critical significant wave height  $H_s^*$ . 'RoPSF' refers to the rate of phase-space flux  $*$   $\Omega$ Figure<br>compa<br>flux.



wave period (s)<br>Figure 10. The significant wave height at  $4^{\circ}$  heel corresponding to equal level of phase-space flux<br>compared with the critical significant wave height  $H^*$  'BoPSE' refers to the rate of phase-space Figure 10. The significant wave height at  $4^{\circ}$  heel corresponding to equal level of phase-space flux compared with the critical significant wave height  $H_{\rm s}^*$ . 'RoPSF' refers to the rate of phase-space flux  $*$   $\Omega$ Figure<br>compa<br>flux.

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wave period (s)<br>Figure 11. The significant wave height at  $8^{\circ}$  heel corresponding to equal level of phase-space flux<br>compared with the critical significant wave height  $H^*$  'BoPSE' refers to the rate of phase-space Figure 11. The significant wave height at  $8^{\circ}$  heel corresponding to equal level of phase-space flux compared with the critical significant wave height  $H_{\rm s}^*$ . 'RoPSF' refers to the rate of phase-space flux  $*$   $\mathbf{Q}$ Figure<br>compa<br>flux.  $4.$  The effects of the damping model on the critical wave height

4. The effects of the<br>Similar calculations for  $H_s^*$  has linear and quadratic the damping model on the critical wave height<br>  $\frac{1}{s}$  have been previously carried out for the system with the<br>  $\frac{1}{s}$  damping terms i.e. equation (2.9) (Jiang et al. 1996) 4. The enects of the damping moder on the critical wave height<br>Similar calculations for  $H_s^*$  have been previously carried out for the system with the<br>usual linear and quadratic damping terms, i.e. equation (2.9) (Jiang Similar calculations for  $H_s^*$  have been previously carried out for the system with the usual linear and quadratic damping terms, i.e. equation  $(2.9)$  (Jiang *et al.* 1996).<br>However, as mentioned before, there is some u usual linear and quadratic damping terms, i.e. equation  $(2.9)$  (Jiang *et al.* 1996).<br>However, as mentioned before, there is some uncertainty in the linear damping coef-<br>ficient when these constant coefficient models are However, as mentioned before, there is some uncertainty in the linear damping coef-<br>ficient when these constant coefficient models are employed, since no single frequency<br>is present in the response. The range of possible ficient when these constant coefficient models are employed, since no single frequency<br>is present in the response. The range of possible values includes those at the linear<br>natural roll frequency  $\omega_n$ , or a typical excit natural roll frequency  $\omega_n$ , or a typical excitation frequency, such as the characteristic wave frequency  $\omega_z$ , but neither are the correct representation. Using the memorynatural roll frequency  $\omega_n$ , or a typical excitation frequency, such as the characteristic<br>wave frequency  $\omega_z$ , but neither are the correct representation. Using the memory-<br>dependent model, one can determine the import wave frequency  $\omega_z$ , but neither are the correct representation. Using the memory-<br>dependent model, one can determine the importance of the linear damping model<br>employed, and the effect that various approximations and as dependent model, one can determine the importance of the linear damping model<br>employed, and the effect that various approximations and assumptions will have on<br>the prediction of system behaviour in terms of capsize probabi employed, and the effect that various approximations and assumptions will have on<br>the prediction of system behaviour in terms of capsize probability. These results can<br>be used to determine the most appropriate value of the the prediction of system behaviour in terms of capsize probability. These the used to determine the most appropriate value of the frequency for evalculated equivalent linear damping coefficient in the constant coefficient used to determine the most appropriate value of the frequency for evaluating the uivalent linear damping coefficient in the constant coefficient models.<br>To investigate these issues, we calculated  $H_s^*$  for three cases of

equivalent linear damping coefficient in the constant coefficient models.<br>To investigate these issues, we calculated  $H_s^*$  for three cases of constant frequency:<br> $B_{44} = 0.0$ ,  $B_{44} = B_{44}(\omega_n)$  and  $B_{44} = B_{44}(\omega_z)$ , and To investigate these issues, we calculated  $H_s^*$  for three cases of constant frequency:<br>  $B_{44} = 0.0$ ,  $B_{44} = B_{44}(\omega_n)$  and  $B_{44} = B_{44}(\omega_z)$ , and the results were compared with<br>
those from the memory function model. Th  $B_{44} = 0.0$ ,  $B_{44} = B_{44}(\omega_n)$  and  $B_{44} = B_{44}(\omega_z)$ , and the results were compared with those from the memory function model. The results are depicted in figures 12, 13 and 14 for heel angles of 0, 4 and 8°, respectivel ∪ those from the memory function model. The results are depicted in figures 12, 13 ○ and 14 for heel angles of 0, 4 and 8°, respectively. The linear damping model, which <br>
∽ uses the memory function, accounts for 16% of and 14 for heel angles of 0, 4 and 8°, respectively. The linear damping model, which<br>uses the memory function, accounts for 16% of the total damping effect in the unbi-<br>ased case, as shown in table 2. That is, the differe uses the memory function, accounts for 16% of the total damping<br>ased case, as shown in table 2. That is, the difference between<br>function and  $H_s^*$  with no linear damping is 16% of the final  $H_s^*$  w<br>the memory function m mping effect in the unbi-<br>en  $H_s^*$  with the memory<br> $s^*$  value. The influence of<br> $s$  to 27% for 4° heel, and ased case, as shown in table 2. That is, the difference between  $H_s^*$  with the memory<br>function and  $H_s^*$  with no linear damping is 16% of the final  $H_s^*$  value. The influence of<br>the memory function model is increased i function and  $H_s^*$  with no linear damping is 16% of the final  $H_s^*$  value. The influence of<br>the memory function model is increased in the biased systems, to 27% for 4° heel, and<br>to 34% for 8° heel. Similar conclusions a the memory function model is increased in the biased systems, to 27% for 4° heel, and<br>to 34% for 8° heel. Similar conclusions are found to hold for the model that assumes<br> $B_{44} = B_{44}(\omega_n)$ , where it represents 15, 18 and to 34% for 8° heel. Similar conclusions are found to hold for the model that assumes  $B_{44} = B_{44}(\omega_n)$ , where it represents 15, 18 and 26% of the total damping for 0, 4 and 8° heel angles, respectively. Similarly, for the and 8° heel angles, respectively. Similarly, for the model that assumes  $B_{44} = B_{44}(\omega_z)$ ,<br>*Phil. Trans. R. Soc. Lond.* A (2000)

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wave period (s)<br>Figure 12. The influence of linear damping modelling on the critical significant wave height  $H_s^*$ <br>in zero heel,  $B_{14}(0.587)$  corresponds to  $\omega = \omega$ ,  $B_{14}(0.700)$  corresponds to a typical excitation ¤ s Figure 12. The influence of linear damping modelling on the critical significant wave height  $H_s^*$  in zero heel.  $B_{44} (0.587)$  corresponds to  $\omega = \omega_n$ .  $B_{44} (0.700)$  corresponds to a typical excitation frequency  $\omega = 0$ Figure 12. The influence<br>in zero heel.  $B_{44}(0.587)$ <br>frequency  $\omega = 0.700$ .

frequency  $\omega = 0.700$ .<br>the results are 23, 28 and 37% for 0, 4 and 8<sup>°</sup> heel angles, respectively. It is seen that, the results are 23, 28 and 37% for 0, 4 and 8° heel angles, respectively. It is seen that, for the vessel under consideration, the model using  $B_{44} = B_{44}(\omega_n)$  underestimates the effects of linear damping, leading to con the results are 23, 28 and 37% for 0, 4 and 8° heel angles, respectively. It is seen that,<br>for the vessel under consideration, the model using  $B_{44} = B_{44}(\omega_n)$  underestimates<br>the effects of linear damping, leading to con for the vessel under consideration, the model using  $B_{44} = B_{44}(\omega_n)$  underestimates<br>the effects of linear damping, leading to conservative safety predictions. On the other<br>hand, the  $B_{44} = B_{44}(\omega_z)$  model slightly overe the effects of linear damping, leading to conservative safety predictions. On the other<br>hand, the  $B_{44} = B_{44}(\omega_z)$  model slightly overestimates the damping effect, resulting<br>in an underestimation of the actual likelihood hand, the  $B_{44} = B_{44}(\omega_z)$  model slightly overestimates the damping effect, resulting<br>in an underestimation of the actual likelihood of capsize. There is no reason to expect<br>that these trends will be universal. If the co in an underestimation of the actual likelihood of capsize. There is no reason to expect<br>that these trends will be universal. If the constant coefficient model is to be used<br>for ship stability studies, the 'best' value for that these trends will be universal. If the constant coefficient model is to be for ship stability studies, the 'best' value for  $B_{44}$  will depend on the ship's if requency, the ship geometry and the characteristics of frequency, the ship geometry and the characteristics of the wave spectrum.<br>The  $B_{44} = B_{44}(\omega_n)$  model can approximately achieve the same damping effect

frequency, the ship geometry and the characteristics of the wave spectrum.<br>The  $B_{44} = B_{44}(\omega_n)$  model can approximately achieve the same damping effect<br>as the memory function model and causes only small error in  $H_s^*$  i The  $B_{44} = B_{44}(\omega_n)$  model can approximately achieve the same damping effect<br>as the memory function model and causes only small error in  $H_s^*$  in the unbiased<br>case (see figure 12). However, when the vessel heels, the  $B$ underestimates  $H_s^*$ . In contrast, the  $B_{44} = B_{44}(\omega_z)$  model overestimates  $H_s^*$  for<br>the unbiased system (it yields almost double the actual linear damping effect), but<br>closely matches the memory function result for t nction model and causes only small error in  $H_s^*$  in the unbias 2). However, when the vessel heels, the  $B_{44} = B_{44}(\omega_n)$  model  $\omega_n^*$ . In contrast, the  $B_{44} = B_{44}(\omega_z)$  model overestimates  $H_s^*$  is the m (it yields a case (see figure 12). However, when the vessel heels, the  $B_{44} = B_{44}(\omega_n)$  model<br>underestimates  $H_s^*$ . In contrast, the  $B_{44} = B_{44}(\omega_z)$  model overestimates  $H_s^*$  for<br>the unbiased system (it yields almost double the a the unbiased system (it yields almost double the actual linear damping effect), but closely matches the memory function result for the two biased cases considered. It is anticipated that, for other heel angles,  $B_{44}$  at closely matches the memory function result for the two biased cases considered. It<br>is anticipated that, for other heel angles,  $B_{44}$  at other frequencies would offer better<br>estimates for  $H_s^*$ . It must be concluded tha

## For  $H_s^*$ . It must be concluded that, in general, there is no 'best' from 5.<br>5. Simulations of the integro-differential equations (*a*) *The simulation model*

 $(a)$  *The simulation model*<br>In this section, time-domain simulation is used to calibrate the capsize probability In this section, time-domain simulation is used to calibrate the capsize probability<br>at the critical significant wave height  $H_s^*$ . Random time histories (realizations) of<br>the roll moment are generated from the spectrum In this section, time-domain simulation is used to calibrate the capsize probability<br>at the critical significant wave height  $H_s^*$ . Random time histories (realizations) of<br>the roll moment are generated from the spectrum at the critical significant wave height  $H_s^*$ . Random time histories (realizations) of the roll moment are generated from the spectrum (equation  $(2.2)$ ) by the scheme proposed by Cuong *et al.* (1982). The equations of *Phil. Trans. R. Soc. Lond.* A (2000)

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wave period (s)<br>Figure 13. The influence of linear damping modelling on the critical significant wave height  $H_s^*$ <br>in  $A^{\circ}$  heel,  $B_{tt}(0.587)$  corresponds to  $\omega = \omega_k - B_{tt}(0.700)$  corresponds to a typical excitation ¤ s Figure 13. The influence of linear damping modelling on the critical significant wave height  $H_s^*$  in 4° heel.  $B_{44} (0.587)$  corresponds to  $\omega = \omega_n$ .  $B_{44} (0.700)$  corresponds to a typical excitation frequency  $\omega = 0.7$ Figure 13. The influend<br>in 4° heel.  $B_{44}$  (0.587)<br>frequency  $\omega = 0.700$ .



wave period (s)<br>Figure 14. The influence of linear damping modelling on the critical significant wave height  $H_s^*$ <br>in  $S^{\circ}$  hool,  $B_U(0.587)$  corresponds to  $\omega = \omega - B_U(0.700)$  corresponds to a typical oxcitation Figure 14. The influence of linear damping modelling on the critical significant wave height  $H_s^*$ Figure 14. The influence of linear damping modelling on the critical significant wave height  $H_s^*$  in 8° heel.  $B_{44} (0.587)$  corresponds to  $\omega = \omega_n$ .  $B_{44} (0.700)$  corresponds to a typical excitation frequency  $\omega = \omega =$ Figure 14. The influence of I<br>in 8° heel.  $B_{44}$  (0.587) corres<br>frequency  $\omega = \omega_z = 0.700$ .

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Table 2. The asymptotic significant wave heights for different linear damping models

**MATHEMATICAL,<br>PHYSICAL<br>& ENGINEERING<br>SCIENCES** Table 2. The asymptotic significant wave heights for different linear damping models<br>(The characteristic wave period is 9.0 s for all cases.  $B_{44}(\omega_n) = 0.321 \times 10^4 \text{ kg m}^{-2} \text{ s}^{-1}$ ,<br> $B_{44}(\omega_n) = 0.548 \times 10^4 \text{ kg m}^{-2} \text{$ Table 2. The asymptotic significant wave heights for all cases.<br>
(The characteristic wave period is 9.0 s for all cases.  $B_{44}(\omega_n) = 0.321 \times B_{44}(\omega_z) = 0.548 \times 10^4 \text{ kg m}^{-2} \text{ s}^{-1}$ ,  $\omega_n = 0.587 \text{ rad s}^{-1}$ ,  $\omega_z = 0.700 \text{ rad$ 



 $A_4 = 0.0$ <br>are then integrated for an exposure time of up to 34.1 min. Capsize is defined as the<br>roll angle exceeding the angle of vanishing stability at any time during the exposure are then integrated for an exposure time of up to 34.1 min. Capsize is defined as the roll angle exceeding the angle of vanishing stability at any time during the exposure period. For each significant wave height and chara are then integrated for an exposure time of up to 34.1 min. Capsize is defined as the roll angle exceeding the angle of vanishing stability at any time during the exposure period. For each significant wave height and chara Foll angle exceeding the angle of vanishing stability at any time during the exposure<br>period. For each significant wave height and characteristic wave period combination,<br>500 realizations are used. Each realization is base period. For each significant wave height and characteristic wave period combination,<br>500 realizations are used. Each realization is based on a statistically equivalent but<br>temporally different roll-excitation time history. 500 realizations are use<br>temporally different rol<br>are set equal to zero.<br>For the usual roll d temporally different roll-excitation time history. The initial roll angle and roll velocity<br>are set equal to zero.<br>For the usual roll differential equation, i.e. equation (2.1), there are no special

numerical difficulties. However, integration of the history-dependent integro-differen-For the usual roll differential equation, i.e. equation  $(2.1)$ , there are no special<br>numerical difficulties. However, integration of the history-dependent integro-differen-<br>tial system is not so straightforward, due to t numerical difficulties. However, integration of the history-dependent integro-differen-<br>tial system is not so straightforward, due to the convolution integral. There exist<br>several methods in the literature to numerically tial system is not so straightforward, due to the convolution integral. There exist<br>several methods in the literature to numerically integrate the integro-differential<br>equation, i.e. equations (2.4) or (2.8), depending ho handled. uation, i.e. equations  $(2.4)$  or  $(2.8)$ , depending how the convolution integral is ndled.<br>The direct integration method is used by Takagi *et al.*  $(1984)$  in their study of options of moored bodies. This is effective s

handled.<br>The direct integration method is used by Takagi *et al.* (1984) in their study of motions of moored bodies. This is effective since the impulse response function is non-zero over a relatively short period (figure The direct integration method is used by Takagi *et al.* (1984) in their study of motions of moored bodies. This is effective since the impulse response function is non-zero over a relatively short period (figure 2) and d motions of moored bodies. This is effective since the impulse response function is<br>non-zero over a relatively short period (figure 2) and direct integration of the convo-<br>lution integral is quick. If the memory effects spa non-zero over a relati<br>lution integral is quic<br>becomes inefficient.<br>Another approach Fraction integral is quick. If the memory effects span a large time interval, this method<br>comes inefficient.<br>Another approach to this class of systems is to use an augmented state space (see,<br>rexample McCreight 1986: Jian

*IATHEMATICAL,<br>'HYSICAL<br>k ENGINEERING<br>CIENCES* for example, McCreight 1986; Jiang *et al.* 1987; Holappa & Falzarano 1999). Such for example, McCreight 1986; Jiang *et al.* 1987; Holappa & Falzarano 1999). Such Another approach to this class of systems is to use an augmented state space (see, for example, McCreight 1986; Jiang *et al.* 1987; Holappa & Falzarano 1999). Such an approach is often quite useful, and in this study it for example, McCreight 1986; Jiang *et al.* 1987; Holappa & Falzarano 1999). Such an approach is often quite useful, and in this study it provides a relatively simple means of justifying the application of the planar Meln an approach is often quite useful, and in this study it provides a relatively simple<br>means of justifying the application of the planar Melnikov method (see appendices A<br>and B). This method uses an augmented state space to means of justifying the application of the planar Melnikov method (see appendices A and B). This method uses an augmented state space to approximate the convolution integral by a system of  $n$  linear ODEs whose transfer f and B). This method uses an augmented state space to approximate the convolution<br>integral by a system of  $n$  linear ODEs whose transfer function can be constructed<br>from the linear damping and added mass coefficients in a integral by a system of *n* linear ODEs whose transfer function can be constructed<br>from the linear damping and added mass coefficients in a systematic, but not unique,<br>manner (Warwick 1989). In this way, evaluation of the from the linear damping and added mass coefficients in a systematic, but not unique,<br>manner (Warwick 1989). In this way, evaluation of the convolution integral is avoided<br>at the expense of expanding the original system of manner (Warwick 1989). In this way, evaluation of the convolution integral is avoided<br>at the expense of expanding the original system of two ODEs into  $n + 2$  first-order<br>ODEs. However, the new  $n + 2$  equations are only an at the expense of expanding the original system of two ODEs into  $n + 2$  first-order ODEs. However, the new  $n + 2$  equations are only an approximation of the real system, hopefully converging with large n. Such a procedure ODEs. However, the new  $n + 2$  equations are only an approximation of the real system, hopefully converging with large *n*. Such a procedure is described in detail in Appendix B.<br>In this work, we use direct integration, si system, hopefully converging with large n. Such a procedure is described in detail in

Appendix B.<br>In this work, we use direct integration, since the system memory is  $ca$ . 8 s (figure 2).<br>The computation time of solving the integro-differential equations in this manner is<br>approximately only twice that of us In this work, we use direct integration, since the system memory is  $ca$ . 8 s (figure 2).<br>The computation time of solving the integro-differential equations in this manner is<br>approximately only twice that of using the cons The computation time of solving the integro-differential equations in this manner is<br>approximately only twice that of using the constant coefficient equations, i.e. equa-<br>tion (2.8). For direct simulation, a fourth-order R approximately only twice that of using the constant coefficient equations, i.e. equation  $(2.8)$ . For direct simulation, a fourth-order Runge-Kutta method is employed.<br>Errors may arise from the usual round-off and from th function. The usual round-off and from the truncation of the memory<br>function. The usual round-off errors are dealt with by choosing an appropriate step<br>size, which was found to be about 150 time-steps per characteristic wa Errors may arise from the usual round-off and from the truncation of the memory function. The usual round-off errors are dealt with by choosing an appropriate step size, which was found to be about 150 time-steps per chara size, which was found to be about 150 time-steps per characteristic wave period.<br>Sample simulation results show that the length of the memory function is not of

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Figure 15. Comparison between simulation and theoretical results for  $0^{\circ}$  heel angle with memory effect being accounted for. The integration time-step is 0.0625 s.

primary importance in estimating the probability of capsize, provided that its most primary importance in estimating the probability of capsize, prosignificant part, i.e. the first  $8 \text{ s}$ , is not discarded (Jiang 1995). (*b*) *The results of simulation*

 $(b)$  The results of simulation<br>The results of simulations, including memory effects, are summarized in figures 15, The results of simulations, including memory effects, are summarized in figures 15,<br>16 and 17, which show a comparison of capsize probability with the prediction of<br>Melnikov analysis. The probability of capsize around the The results of simulations, including memory effects, are summarized in figures 15, 16 and 17, which show a comparison of capsize probability with the prediction of Melnikov analysis. The probability of capsize around the 16 and 17, which show a comparison of capsize probability with the prediction of Melnikov analysis. The probability of capsize around the critical significant wave height curve is in the range of 3–7, 5–9 and 1–4% for the lnikov analysis. The probability of capsize around the critical significant wave<br>ght curve is in the range of  $3-7$ ,  $5-9$  and  $1-4\%$  for the unbiased,  $-4^{\circ}$  heel and<br> $\circ$  heel cases, respectively. As can be seen from height curve is in the range of 3–7, 5–9 and  $1-4\%$  for the unbiased,  $-4^{\circ}$  heel and  $-8^{\circ}$  heel cases, respectively. As can be seen from these figures, the theoretical asymptote qualitatively matches the results of  $-8^{\circ}$  heel cases, respectively. As can be seen from these figures, the theoretical asymptote qualitatively matches the results of the simulations quite well. The likelihood of capsize, as predicted by the theoretical c tote qualitatively matches the results of the simulations quite well. The likelihood<br>of capsize, as predicted by the theoretical critical curves, varies from the simulation<br>results by 1–9% for the heel angles and exposure of capsize, as predicted by the theoretical critical curves, varies from the simulation results by  $1-9\%$  for the heel angles and exposure times under consideration. However, the relative ease of computing this curve, wh

results by 1–9% for the heel angles and exposure times under consideration. How-<br>ever, the relative ease of computing this curve, when compared against the extensive<br>nature of the stochastic simulations, strongly recommend ever, the relative es<br>nature of the stoch<br>engineering tool.

### 6. Conclusions

**6. Conclusions**<br>On a general level, this work has shown that analysis tools from the theory of dynam-<br>ical systems can be used to examine nonlinear systems with memory and random In a general level, this work has shown that analysis tools from the theory of dynamical systems can be used to examine nonlinear systems with memory and random excitation. The annihications of the Melnikov function and ph On a general level, this work has shown that analysis tools from the theory of dynamical systems can be used to examine nonlinear systems with memory and random excitation. The applications of the Melnikov function and pha ical systems can be used to examine nonlinear systems with memory and random<br>excitation. The applications of the Melnikov function and phase-space transport tech-<br>niques are used to predict extreme responses that lead to f excitation. The applications of the Melnikov function and phase-space transport techniques are used to predict extreme responses that lead to failure. The methodology is quite general and can be applied to expanded multi-d niques are used to predict extreme responses that lead to failure. The methodology<br>is quite general and can be applied to expanded multi-degree-of-freedom systems.<br>There are no restrictions on the nature of the memory func is quite general and can be applied to expanded multi-degree-of-freedom systems.<br>There are no restrictions on the nature of the memory function, other than it needs<br>to be modelled by a linear integro-differential equation,

 $\frac{1}{6}$  model. be modelled by a linear integro-differential equation, or an expanded state space<br>odel.<br>In this work, these methods have been applied to the capsizing of ships in a ran-<br>m seaway Closed-form asymptotic expressions for the

model.<br>In this work, these methods have been applied to the capsizing of ships in a random seaway. Closed-form asymptotic expressions for the rate of phase-space flux and dom seaway. Closed-form asymptotic expressions for the rate of phase-space flux and<br>*Phil. Trans. R. Soc. Lond.* A (2000)

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characteristic wave period (s)<br>Figure 16. Comparison between simulation and theoretical results for  $-4^{\circ}$  heel angle with<br>memory effect being accounted for The integration time-step is 0.0625 s 6. Comparison between simulation and theoretical results for  $-4^{\circ}$  heel angimemory effect being accounted for. The integration time-step is 0.0625 s.



characteristic wave period (s)<br>Figure 17. Comparison between simulation and theoretical results for  $-8^{\circ}$  heel angle with<br>memory effect being accounted for. The integration time-step is 0.0625 s 7. Comparison between simulation and theoretical results for  $-8^{\circ}$  heel angimemory effect being accounted for. The integration time-step is 0.0625 s.

memory effect being accounted for. The integration time-step is 0.0625 s.<br>an associated critical significant wave height are derived and evaluated using simulations. Using these results, it was found that the memory function influences the an associated critical significant wave height are derived and evaluated using simulations. Using these results, it was found that the memory function influences the probability of capsize. And, while one can replace the m lations. Using these results, it was found that the memory function influences the probability of capsize. And, while one can replace the memory function with constant added mass and damping coefficients evaluated at a sin probability of capsize. And, while one can replace the memory function with constant added mass and damping coefficients evaluated at a single frequency, a general relation for the determination of the 'best' frequency cou stant added mass and damping coefficients evaluated at a single frequency, a general<br>relation for the determination of the 'best' frequency could not be found. Depend-<br>ing upon vessel conditions (heel angle and mass distri relation for the determination of the 'best' frequency could not be found. Depending upon vessel conditions (heel angle and mass distribution) and sea environment (significant wave height and characteristic wave period), d ing upon vessel conditions (heel angle and mass distribution) and sea environment<br>(significant wave height and characteristic wave period), different strategies for the<br>selection of the 'best' frequency may be possible. It (significant wave height and characteristic wave period), different strategies for the selection of the 'best' frequency may be possible. It is expected that none of these strategies is universal. As such, it is recommende selection of the 'best' frequency may be possible. It is expected that none of these strategies is universal. As such, it is recommended that the hydrodynamic coefficients include the influence of memory effects.

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Capsize criteria with memory-aepenaent models [185]<br>This paper has been sponsored by the Michigan Sea Grant College Program, Projects R/T-<br>32 and R/T-37, under grant numbers NA89AA-D-SG083 Amd#5 and DOC-G-NA76RG0133 This paper has been sponsored by the Michigan Sea Grant College Program, Projects R/T-32 and R/T-37, under grant numbers NA89AA-D-SG083 Amd#5 and DOC-G-NA76RG0133<br>from the Office of Sea Grant National Oceanic and Atmospher This paper has been sponsored by the Michigan Sea Grant College Program, Projects R/T-<br>32 and R/T-37, under grant numbers NA89AA-D-SG083 Amd#5 and DOC-G-NA76RG0133<br>from the Office of Sea Grant, National Oceanic and Atmosph 32 and R/T-37, under grant numbers NA89AA-D-SG083 Amd#5 and DOC-G-NA76RG0133<br>from the Office of Sea Grant, National Oceanic and Atmospheric Administration (NOAA), US<br>Department of Commerce, and funds from the State of Mich Department of Commerce, and funds from the State of Michigan. The government is authorized to produce and distribute reprints for government purpose notwithstanding any copyright<br>notation appearing herein. S.W.S. also acknowledges partial support from the National Science<br>Foundation and helpful discussions notation appearing herein. S.W.S. also acknowledges partial support from the National Science

# Foundation and helptul discussions with Hassan Khalil on linear system dynamics.<br>Appendix A. The Melnikov function for systems with memory effects

Appendix A. The Melnikov function for systems with memory effects<br>In this appendix we provide the technical justification for the Melnikov function used<br>in the paper, and for the use of phase-space transport ideas for this In this appendix we provide the technical justification for the Melnikov function used<br>in the paper, and for the use of phase-space transport ideas for this class of systems.<br>This is necessary because the perturbation term In this appendix we provide the technical justification for the Melnikov function used<br>in the paper, and for the use of phase-space transport ideas for this class of systems.<br>This is necessary because the perturbation term in the paper, and for the use of phase-space transport ideas for this class of systems.<br>This is necessary because the perturbation terms in the integro-differential equation<br>are not simply dependent on the instantaneous st This is necessary because the perturbation terms in the integro-differential equation are not simply dependent on the instantaneous states of the system and the forcing  $\bullet$  phase; they also depend on the history of the states. Therefore, the perturbed phase is actually infinite dimensional. In this derivation, we will use an expanded state space model to approximate the memory effects. space is actually infinite dimensional. In this derivation, we will use an expanded state space model to approximate the memory effects. We begin by approximating the memory term as the output of a finite-dimensional dynam space is actually infinite dimensional. In this derivation, we will use an expanded<br>state space model to approximate the memory effects. We begin by approximating the<br>memory term as the output of a finite-dimensional dynam state space model to approximate the memory effects. We begin by approximating the<br>memory term as the output of a finite-dimensional dynamical system. The procedure<br>for accomplishing this is described in detail in Appendix memory term as the output of a finite-dimensional dynamical system. The procedure<br>for accomplishing this is described in detail in Appendix B. We then use the general<br>structure of these systems to derive the appropriate Me for accomplishing this is described in detail in Appendix B. We then use the general structure of these systems to derive the appropriate Melnikov function and provide an interpretation of the phase-space transport. The ke structure of these systems to derive the appropriate Melnikov function and provide<br>an interpretation of the phase-space transport. The key step to the approach taken an interpretation of the phase-space transport. The key step to the approach<br>is to show that the dynamics of this high-dimensional system really take place<br>low-dimensional stable invariant manifold, on which the usual resu to show that the dynamics of this high-dimensional system really take place in a<br>w-dimensional stable invariant manifold, on which the usual results apply.<br>The memory term can be approximated by the output of some finite-d

low-dimensional stable invariant manifold, on which the usual results apply.<br>The memory term can be approximated by the output of some finite-dimensional linear system, as follows. First, denote

First, denote  
\n
$$
z_1(y,t) = \int_{-\infty}^{\infty} \delta_m(t-u)y(u) \, \mathrm{d}u \tag{A1}
$$

 $z_1(y,t) = \int_{-\infty}^{t} \delta_m(t-u)y(u) \, \mathrm{d}u$  (A 1)<br>and let  $z_1(y,t)$  be the first element of a vector  $z(y,t)$ , whose dynamics are governed<br>by a linear system of the form and let  $z_1(y, t)$  be the first element c<br>by a linear system of the form  $\dot{z} = Az + by,$  (A 2)

$$
\dot{\mathbf{z}} = \mathbf{A}\mathbf{z} + \mathbf{b}\mathbf{y},\tag{A.2}
$$

 $\dot{z} = Az + by,$  (A 2)<br>where the (scalar) roll velocity y plays the role of an input. In this setting, the<br>equations of motion in first-order form are given by where the (scalar) roll velocity  $y$  plays the role of an equations of motion, in first-order form, are given by equations of motion, in first-order form, are given by

$$
\dot{x}(t) = y(t),
$$
  
\n
$$
\dot{y}(t) = f(x) + \epsilon g(x, y, cz, t),
$$
  
\n
$$
\dot{z} = Az + by,
$$

Where the specific forms of  $f(x)$ ,  $g(x, y, cz, t)$ , **A** and **b** can be found by comparison where the specific forms of  $f(x)$ ,  $g(x, y, cz, t)$ , **A** and **b** can be found by comparison<br>with the equations given in Appendix B, and **c** is simply a vector that picks off the<br>first element of z. Our goal here is to show tha where the specific forms of  $f(x)$ ,  $g(x, y, cz, t)$ , **A** and **b** can be found by comparison<br>with the equations given in Appendix B, and **c** is simply a vector that picks off the<br>first element of z. Our goal here is to show tha with the equations given in Appendix B, and c is simply a vector that picks off the first element of z. Our goal here is to show that the first two equations correctly capture the full dynamics, at least in some invariant So first element of z. Our goal here is to show that the first two equations correctly capture the full dynamics, at least in some invariant manifold of the  $(x, y, z)$  phase space, by replacing z with an appropriate functio In that case, the usual Melnikov theory goes through, as does the interpretation of hy phase-space transport for periodically forced oscillators (Wiggins 1992).

In the unperturbed system (i.e.  $\epsilon = 0$ ), the  $(x, y)$  dynamics are uncoupled from the *<sup>z</sup>* dynamics. For notational purposes, we denote the unperturbed solutions as

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1786 C. Jiang, A. W. Troesch and  $\check{S}$ . W. Shaw<br>  $(x, y, z) = (x_{\delta}, y_{\delta}, z_{\delta})$ . As described in § 5, the  $(x_{\delta}, y_{\delta})$  dynamics are those of the<br>
well-known conservative phase planes shown in figures 3 and 4. Once  $u_{\delta}$  is  $(x, y, z) = (x_{\delta}, y_{\delta}, z_{\delta})$ . As described in § 5, the  $(x_{\delta}, y_{\delta})$  dynamics are those of the well-known conservative phase planes shown in figures 3 and 4. Once  $y_{\delta}$  is known from the  $(x_{\delta}, y_{\delta})$  system, the  $z_{\delta}$  dyn  $(x, y, z) = (x_{\delta}, y_{\delta}, z_{\delta})$ . As described in §5, the  $(x_{\delta}, y_{\delta})$  dynamics are those of the well-known conservative phase planes shown in figures 3 and 4. Once  $y_{\delta}$  is known from the  $(x_{\delta}, y_{\delta})$  system, the  $z_{\delta}$  dyna well-known conservative phase planes shown in figures 3 and 4. Once  $y_{\delta}$  is known<br>from the  $(x_{\delta}, y_{\delta})$  system, the  $z_{\delta}$  dynamics are those of a linear system with known<br>input. The specific solution of interest here from the  $(x_{\delta}, y_{\delta})$  system, the  $z_{\delta}$  dynamics are those of a linear system with known

need and the eigenvalues of **A** are an stable; it is given by  
\n
$$
z_{\delta}(y, t) = \int_{-\infty}^{t} e^{A(t-\tau)} by_{\delta}(\tau) d\tau.
$$
\n(A3)

These solutions for  $\boldsymbol{z}$  represent, in the fully extended  $(x, y, \boldsymbol{z})$  phase space, a two-These solutions for z represent, in the fully extended  $(x, y, z)$  phase space, a two-<br>dimensional invariant manifold of the form  $z = F_0(x, y)$ . Furthermore, since the<br>transient part of the z dynamics is exponentially stable, These solutions for  $z$  represent, in the fully extended  $(x, y, z)$  phase space, a two-<br>dimensional invariant manifold of the form  $z = F_0(x, y)$ . Furthermore, since the<br>transient part of the  $z$  dynamics is exponentially stab fold. For 0  $\times$  6  $\times$  1, the (x, y, z) dynamics become fully coupled. For this system, there<br>For  $0 < \epsilon \ll 1$ , the (x, y, z) dynamics become fully coupled. For this system, there

exists an asymptotically stable, perturbed manifold of the form  $\mathbf{F} = \mathbf{F}_0 + \epsilon \mathbf{F}_1 + \cdots$ . On this manifold, the equations of motion are given by

$$
\begin{aligned}\n\dot{x}(t) &= y(t), \\
\dot{y}(t) &= f(x) + \epsilon g(x, y, c\mathbf{F}_0(x, y), t) + O(\epsilon^2). \\
\end{aligned}\n\tag{A.4}
$$

 $\dot{y}(t) = f(x) + \epsilon g(x, y, c\mathbf{F}_0(x, y), t) + O(\epsilon^2)$ .<br>The Melnikov function for this two-dimensional system is then given by the usual planar result, as follows: The Melnikov function for<br>planar result, as follows:

planar result, as follows:  
\n
$$
M_{\delta}(t_0) = \int_{-\infty}^{\infty} y_{\delta}(t) g(x_{\delta}, y_{\delta}, cz_{\delta}, t + t_0) dt,
$$
\n( A5)  
\nwhere the unperturbed solution is taken along the homoclinic or heteroclinic orbit.  
\nFinally, by identifying the various terms and their roles in the perturbation func-

Finally, by identifying the various terms and their roles in the perturbation func-<br>tion  $g$ , especially the fact that where the unperturbed solution if Finally, by identifying the variation  $g$ , especially the fact that

$$
cz = z_1 = \int_{-\infty}^{\infty} \delta_m(t - u) y(u) \, \mathrm{d}u,
$$
 (A 6)

 $cz = z_1 = \int_{-\infty}^{\infty} o_m(t-u)y(u) du,$  (A b)<br>it can be concluded that the Melnikov function for the system with memory can<br>be well approximated by the expression given in (3.7). This of course depends be concluded that the Melnikov function for the system with memory can<br>be well approximated by the expression given in  $(3.7)$ . This, of course, depends<br>on finding an auxiliary linear system whose output provides a good m it can be concluded that the Melnikov function for the system with memory can<br>be well approximated by the expression given in (3.7). This, of course, depends<br>on finding an auxiliary linear system whose output provides a go be well approximated by the expression given in  $(3.7)$ . This, of course, depends on finding an auxiliary linear system whose output provides a good match for the memory function. That procedure is described in Appendix B.

## memory function. That procedure is described in Appendix B.<br>Appendix B. The extended state space model for the radiation force Appendix B. The extended state space model for the The radiation force is proportional to the convolution integral

The radiation force is proportional to the convolution integral

$$
\beta_1(\tau) = \int_{-\infty}^{+\infty} K(\tau - u)\dot{\phi}(u) du.
$$
 (B1)

 $\beta_1(\tau) = \int_{-\infty}^{\infty} K(\tau - u)\phi(u) du.$  (B1)<br>It is to be approximated as an output of an *n*th-order linear system with  $\dot{\phi}(\tau)$  as its<br>input. The form of this equation is taken to be It is to be approximated as an output of an *n*th-or<br>input. The form of this equation is taken to be

form of this equation is taken to be  
\n
$$
\left[a_{n+1}\frac{d^n}{d\tau^n} + a_n\frac{d^{n-1}}{d\tau^{n-1}} + a_{n-1}\frac{d^{n-2}}{d\tau^{n-2}} + \dots + a_2\frac{d}{d\tau} + a_1\right]\beta_1 = \dot{\phi}.
$$
 (B2)

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As pointed out in  $\S 5$ , there are several linear systems able to model this radiation As pointed out in §5, there are several linear systems able to model this radiation moment, given by different orders and coefficients in the linear differential operator.<br>Taking the Fourier transform of both sides vields As pointed out in §5, there are several linear systems able to model this radiation<br>moment, given by different orders and coefficients in the linear differential operate<br>Taking the Fourier transform of both sides yields t Taking the Fourier transform of both sides yields the transfer function of  $(B2)$ ,

$$
\frac{B_1(\omega)}{\Phi_d(\omega)} = 1 / \sum_{k=1}^{n+1} a_k (\mathrm{i}\omega)^{k-1},
$$
 (B3)

where  $B_1(\omega)$  and  $\Phi_d(\omega)$  are the Fourier transforms of  $\beta_1(\tau)$  and  $\dot{\phi}_d(\tau)$ , respectively.<br>From the Fourier transform of (B1), we obtain where  $B_1(\omega)$  and  $\Phi_d(\omega)$  are the Fourier transforms<br>From the Fourier transform of (B 1), we obtain

(1), we obtain  
\n
$$
\frac{B_1(\omega)}{\Phi_d(\omega)} = 2\pi \mathcal{K}(\omega).
$$
\n(B4)

 $\frac{1}{\Phi_{\rm d}(\omega)} = 2\pi \mathcal{K}(\omega).$  (B4)<br>The Fourier transform of the memory function  $K(\tau)$  is given by (see, for example,<br>Takagi *et al.* 1984) The Fourier transforn<br>Takagi *et al*. 1984)

Taking the following theorem to the identity function 
$$
L(\cdot)
$$
 is given by the function  $K(\omega) = B_{44}(\omega) - B_{44}(\infty) + i\omega(A_{44}(\omega) - A_{44}(\infty)).$  (B5) Comparing these expressions for the transfer function, we obtain the relation

ng these expressions for the transfer function, we obtain the relation  
\n
$$
B_{44}(\omega) - B_{44}(\infty) + i\omega(A_{44}(\omega) - A_{44}(\infty)) = 1 / \sum_{k=1}^{n+1} a_k (i\omega)^{k-1},
$$
\n(B6)

which is equivalent to

uivalent to  
\n
$$
\frac{(B_{44}(\omega) - B_{44}(\infty)) - i\omega(A_{44}(\omega) - A_{44}(\infty))}{\Gamma} = \sum_{k=1}^{n+1} a_k (i\omega)^{k-1},
$$
\n(B7)

where

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$$
\Gamma = (B_{44}(\omega) - B_{44}(\infty))^2 + \omega^2 (A_{44}(\omega) - A_{44}(\infty))^2.
$$
 (B8)

 $\Gamma = (B_{44}(\omega) - B_{44}(\infty))^2 + \omega^2 (A_{44}(\omega) - A_{44}(\infty))^2$ . (B8)<br>By equating the real and imaginary parts of the two sides, we obtain two objective<br>functions that are used for curving fitting. For example, the objective functions By equating the real and imaginary parts of the two sides, we obtain two objective functions that are used for curving fitting. For example, the objective functions for  $n = 8$  are given by By equating the real a functions that are use<br> $n = 8$  are given by

$$
a_9\omega^8 - a_7\omega^6 + a_5\omega^4 - a_3\omega^2 + a_1 = \frac{B_{44}(\omega) - B_{44}(\infty)}{\Gamma},
$$
 (B.9)

$$
a_7\omega^6 + a_5\omega^4 - a_3\omega^2 + a_1 = \frac{1}{\Gamma}, \qquad (B9)
$$
  

$$
a_8\omega^6 - a_6\omega^4 + a_4\omega^2 - a_2 = \frac{A_{44}(\omega) - A_{44}(\infty)}{\Gamma}. \qquad (B10)
$$

It is frequently difficult to find polynomials which accurately fit  $(B 9)$  and  $(B 10)$ , It is frequently difficult to find polynomials which accurately fit (B9) and (B10),<br>especially in the most useful frequency ranges  $((0.0, 2.0)$  rad s<sup>-1</sup> in our case). The<br>results of using the  $x^2$  fitting method (Press It is frequently difficult to find polynomials which accurately fit (B 9) and (B 10), especially in the most useful frequency ranges  $((0.0, 2.0) \text{ rad s}^{-1}$  in our case). The results of using the  $\chi^2$  fitting method (Pres  $2_{\text{fi}}$ especially in the most useful frequency ranges  $((0.0, 2.0) \text{ rad s}^{-1}$  in our case). The results of using the  $\chi^2$  fitting method (Press *et al.* 1992) are shown in figures 18 and 19 and table 3. The results vary signific results of using the  $\chi^2$  fitting method (Press *et al.* 1992) are shown in figures 18 and 19 and table 3. The results vary significantly if different individual standard deviations  $\sigma_1$  are used in different frequenc and 19 and table 3. The results vary significantly if different individual standard deviations  $\sigma_1$  are used in different frequency ranges. In 'curve fit 1', we set  $\sigma_i = 0.2$ , 1.0 and  $\infty$  for  $\omega \in (0.0, 2.0)$ ,  $(2.0,$ 1.0 and  $\infty$  for  $\omega \in (0.0, 2.0), (2.0, 6.0)$  and  $(6.0, \infty)$ , respectively. In 'curve fit 2', we set  $\sigma_i = 0.2, 1.0$  and  $\infty$  for  $\omega \in (0.0, 2.0), (2.0, 3.0)$  and  $(3.0, \infty)$ , respectively. 1.0 and  $\infty$  for  $\omega \in (0.0, 2.0)$ ,  $(2.0, 6.0)$  and  $(6.0, \infty)$ , respectively. In 'curve fit 2', we set  $\sigma_i = 0.2$ , 1.0 and  $\infty$  for  $\omega \in (0.0, 2.0)$ ,  $(2.0, 3.0)$  and  $(3.0, \infty)$ , respectively. Neither of the two 9th-o we set  $\sigma_i = 0.2$ , 1.0 and  $\infty$  for  $\omega \in (0.0, 2.0)$ ,  $(2.0, 3.0)$  and  $(3.0, \infty)$ , respectively.<br>Neither of the two 9th-order polynomials fits the curves very well (see figures 18 and 19) and the  $a_i$  change significant Neither of the two 9th-order polynomials fits the curves very well (see figures 18 and 19) and the  $a_i$  change significantly (table 3). In addition, the term  $1/a_{n+1}$  is of order  $10^{11}$  in 'curve fit 1' (and  $10^7$  in order  $10^{11}$  in 'curve fit 1' (and  $10^7$  in 'curve fit 2') while the other leading coefficients<br>*Phil. Trans. R. Soc. Lond.* A (2000)





Figure 18. Curve fit for  $(B_{44}(\omega) - B_{44}(\infty))/\Gamma$  based on  $\chi^2$  fitting method.<br>in the system's stiff matrix (equation (B 11)) are of order 1. This makes the range of<br>relative magnitudes of the eigenvalues of the expanded in the system's stiff matrix (equation  $(B 11)$ ) are of order 1. This makes the range of relative magnitudes of the eigenvalues of the expanded system unacceptably large, which in turn makes numerical integration difficult in the system's stiff matrix (equation  $(B 11)$ ) are of order 1. This makes the range of relative magnitudes of the eigenvalues of the expanded system unacceptably large, which in turn makes numerical integration difficult relative magnitudes of the eigenvalues of the expanded system unacceptably large, which in turn makes numerical integration difficult, a common problem with this approach. Higher-order polynomials can achieve a better fit, approach. Higher-order polynomials can achieve a better fit, but at some point the expanded system will be impossible to solve numerically. In addition, the cost spent<br>on solving such a large system of equations will eventually exceed the cost of direct<br>integration of the system with the memory function. on solving such a large system of equations will eventually exceed the cost of direct solving such a large system of equations will eventually exceed the cost of direct<br>egration of the system with the memory function.<br>Once the size of the model has been determined and the values of the coefficients<br>a obtain

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where

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$$
\phi_{\rm d} = \dot{\phi}, \qquad b_{44}(\infty) = \frac{B_{44}(\infty)}{I_{44} + A_{44}(\infty)}, \qquad C_{\beta} = \frac{1}{I_{44} + A_{44}(\infty)}
$$

$$
F_{\rm nl}(\tau) = \frac{F(\tau) - B_{44q}(\omega)\phi_{\rm d}|\phi_{\rm d}| - \Delta(C_0 + C_3\phi^3 + \dots)}{I_{44} + A_{44}(\infty)}.
$$

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Figure 19. Curve fit for  $(A_{44}(\omega) - A_{44}(\infty))/\Gamma$  based on  $\chi^2$  fitting method.<br>Table 3. *The deviation of curve fitting results with individual standard variations* Table 3. The deviation of curve fitting results with individual standard variations

	'curve fit $1$ '	'curve fit $2'$	
$a_1$ a <sub>2</sub> $a_3$ $a_4$ $a_5$ a <sub>6</sub> $a_7$ $a_8$	$5.57782 \times 10^{-6}$ $-4.55531 \times 10^{-5}$ $-5.72207 \times 10^{-6}$ $-1.67755\times10^{-5}$ $-2.03768 \times 10^{-7}$ $-1.01694 \times 10^{-6}$ $-9.19187 \times 10^{-9}$ $-1.87369 \times 10^{-8}$	$-8.91987 \times 10^{-9}$ $-5.42838 \times 10^{-5}$ $-2.589\,28\times10^{-5}$ $-3.74155 \times 10^{-5}$ $-1.16985 \times 10^{-5}$ $-8.27478 \times 10^{-6}$ $-2.05076 \times 10^{-6}$ $-5.686\,81\times10^{-7}$	
$a_9$	$-4.30184 \times 10^{-11}$	$-1.13707 \times 10^{-7}$	

This is of the form required for the Melnikov theory of Appendix A, with  $\beta_1 = z_1$ <br>and  $(\phi, \dot{\phi}) = (x, y)$ This is of the form r<br>and  $(\phi, \dot{\phi}) = (x, y)$ .

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